

AN INTRODUCTION TO A NEW CONCEPT:  
THE DYNAMIC STABILITY  
OF  
DISKS FREELY DESCENDING IN A FLUID MEDIA

by

BILLY LEE HIMES, SR.

B. S., Kansas State University of  
Agriculture and Applied Science, 1958

---

A THESIS

submitted in partial fulfillment of the

requirements for the degree

MASTER OF SCIENCE

Department of Chemical Engineering

KANSAS STATE UNIVERSITY  
OF AGRICULTURE AND APPLIED SCIENCE

1960

LD  
2668  
T4  
1960  
455  
c. 2  
Document

## TABLE OF CONTENTS

INTRODUCTION . . . . .	1
A Definition of Dynamic Stability . . . . .	1
Objectives of Study . . . . .	3
Review of Literature . . . . .	4
Illustration of Definition of Dynamic Stability . . . . .	6
PROCEDURE OF MEASUREMENTS . . . . .	9
Outline of Procedure . . . . .	9
Detailed Procedure . . . . .	9
Diagrams of Equipment . . . . .	13
MATHEMATICAL DEFINITION OF DYNAMIC STABILITY . . . . .	36
Initial Calculations on Data from Representative Frame of a Descent . . . . .	36
Definition of Dynamic Stability, $\psi_{s^2_{y.x}}$ and $\psi_{\sqrt{s^2_{y.x}}}$ . . . . .	39
Definition of Dynamic Stability, $\psi_{\text{AREA}}$ . . . . .	40
Diagrams of Dynamic Stability . . . . .	42
GENERAL VISUAL OBSERVATIONS ON THE DESCENT OF DISKS THROUGH TRIETHYLENE GLYCOL AND WATER . . . . .	47
Discussion of Observations and Postulations . . . . .	47
Diagrams of Postulated Types of Motion . . . . .	56
EVALUATION OF DATA AND PRESENTATION OF RESULTS . . . . .	65
DISCUSSION OF RESULTS . . . . .	82
CONCLUSIONS . . . . .	85
RECOMMENDATIONS . . . . .	87
ACKNOWLEDGMENTS . . . . .	88
REFERENCES . . . . .	90

APPENDIX . . . . .	91
Tables . . . . .	92
Triethylene Glycol as the Fluid Media . . . . .	.107
Table of Nomenclature . . . . .	.108
Table of Definitions . . . . .	110
IBM 650 Programs for Computation of Dynamic Stability . . . . .	.112

## INTRODUCTION

### A Definition of Dynamic Stability

In the beginning of time on this earth instability in motion was observable when the first leaf fell from the first plant. As the leaf descended toward the earth it either fluttered, oscillated, revolved, spiraled, or combined all or part of these motions in its erratic descent. This erratic behavior was indicative of the non-uniform velocity of the leaf along a vertical to the earth. Therefore it was exhibiting unstable motion, or dynamic instability.

If the first man had been able to measure the height of the leaf at equal increments of time, dynamic stability might have been acknowledged millions of years ago. Initially the first man might have noticed that the measured heights of the leaf did not decrease the same amount over each equal increment of time. This indicated non-uniform velocity in the vertical direction. The first man might then have compared these measured heights with the comparable heights of a leaf imagined as descending toward the earth with uniform velocity in the vertical direction. From comparing each actual height with each imagined height for the same time of descent he would have found a difference. By then calculating the mean of the summation of these absolute differences he might have called the resultant value dynamic instability. However, he didn't. Dynamic instability had always existed, but it was necessary for man to measure it to verify its existence.

Simply, dynamic instability was the mean deviation of the actual path of the leaf from some imaginary path, described in this study by a least squares line through the actual path. For this imaginary path

no assumptions were made as to the uniformity of the leaf's velocity vectors horizontal to the earth's surface. It was possible for the leaf to follow a curvilinear path in its actual descent and exhibit no dynamic unstability as long as it descended equal increments of vertical distance in equal increments of universal time.

The difference, or deviation, of the actual path from an imaginary path had units of length. Therefore, dynamic unstability had units of length. As the phraseology might indicate, dynamic stability was defined as the reciprocal, or opposite, of dynamic unstability. Therefore, its units were reciprocal length. In order to adequately describe the meaning of dynamic stability it was necessary to define it in mathematical terms; this was presented in the Mathematical Definition of Dynamic Stability.

Many investigators, including Wadell (8), Pettyjohn (6), Squires (7), Wetherall (9), and Becker (1), had observed this behavior in most describable sizes and shapes of particles, but carried the investigation little further than recording these observations and, in some cases, comparing them with conventional concepts of particle behavior in fluids. No attempt had been made to numerically evaluate or to define the unstability of movement of particles in fluids.

Disks which exhibit infinite stability are illustrated in PLATE I. They were defined as a stationary disk, floating on the surface of a fluid, with no velocity vector and as a moving disk, descending or ascending in the fluid, with velocity but no acceleration. Conversely, infinite unstability was postulated as the motion of a disk which exhibited constant acceleration through infinite time. A disk accelerating and decelerating through finite time had a finite value of dynamic stability

and instability. It was these latter descents of free-falling disks with which this study was concerned.

Even as the best measurement is no better than an approximation of the true value, so no motion is absolutely dynamically stable. Due to limitations on man's ability to measure minute differences, it is possible to measure and subsequently calculate a descent as dynamically stable, although theoretically a dynamically stable descent is unobtainable. As man's abilities to measure improve that which was previously classified as dynamically stable may no longer be so classified.

#### Objectives of Study

The original objectives of this study were:

- 1) To continue work initiated by Professor R. C. Hall\* and to refine the mathematical definition of the concept of free-falling particle behavior in an infinite expanse of fluid.
- 2) To calculate numerical values of stability from actual descents of disks of varying properties.

Later objectives were:

- 3) To observe and define the types of motion of free-falling disks in a fluid media.
- 4) To correlate these values of dynamic stability with the types of motion observed.

Objectives 3) and 4) were planned after an initial study of the types of motions of disks had shown that some of the disks consistently fell in the same manner.

\*Private communications

## Review of Literature

A literature search revealed that dynamic stability had aroused the curiosity of many investigators, but that this effort was apparently the first actually measuring it. The literature search included the following material:

1. Chemical Abstracts dating back to 1930.
2. Physics Abstracts back to 1950.
3. All theses and abstracts of theses located in the Kansas State University Library.
4. Numerous more recent publications not yet abstracted.

No experimentation resembling dynamic stability studies was uncovered. However, several articles discussing the appearance of motion in particles descending through a fluid media were available.

From an article by Wadell (8), the following was quoted:

Franz Schulze stated that the translation velocity of a flat stone in air or water depends upon its position in respect to the direction of translation. Jordan noted that mineral particles readily took a position with their broad side at right angles to the direction of fall. Schmiedel found that thin disks at very low  $Re$  showed no tendency to take any particular settling position while at higher  $Re$  the disks tended toward a horizontal position regardless of the position at the start. At still higher  $Re$ 's periodic oscillations set in and the settling took place in a zig-zag path.

On the study of spheres, octahedrons, cubes, and tetrahedrons Pettyjohn and Christiansen (6), stated:

Whereas in the streamline flow range ( $Re < 0.05$ ) the particles did not favor any particular orientation with respect to the direction of motion, at  $Re \sim 10$ , the tetrahedrons and cubes assumed an orientation with a face in the horizontal plane or perpendicular to the direction of motion and the other particles showed a tendency to do likewise by  $Re \sim 20$ . This orientation was maintained up to  $Re = 70$  to 300 where the particles beginning with the tetrahedrons, 'teetered' or

wobbled and eventually spun or rolled on a horizontal axis and followed a spiral path rather than a straight vertical path in their descent.

From an article on the sedimentation of thin disks by Squires and Squires (7), it was stated that at very low Reynolds numbers, in the viscous region a body, such as a disk, possessing three perpendicular planes has no tendency to assume any particular orientation during fall. It was further stated:

A disc placed with its flat faces parallel to the direction of motion of the fluid will maintain this orientation. At higher Reynolds numbers, in the turbulent region, the eddies set up by the motion of the disc through the fluid act to maintain the particle at right angles to the direction of motion. Therefore, a disc will always settle in a horizontal position in the turbulent region, and its position when in the viscous region will depend upon its initial inclination.

Miyagi (5), discussed the motion of an air bubble rising in water. The course of the bubble was three-dimensional; it was observed to be a helix around a vertical. The major axis of the bubble was always perpendicular to its course. PLATE XVIII, Fig. 1, based on Fig. 12 of Miyagi's article illustrated the rise of an air bubble in only two of the three dimensions.

Withersall (9), was able to photograph the track of a flat solid (a small crystal of  $\text{AgNO}_3$ ) as it descended in acidulated water. PLATE XVIII, Fig. 2, was based on this photograph.

Bescker (1), provided the following information in a table on the inertial drag characteristics of freely oriented bluff bodies. At  $Re$ 's of 0.1 - 5.5 it was stated that all orientations were stable when there were three or more perpendicular axes of symmetry in the descending disk. At  $Re$ 's of 5.5 - 200 the motion was stable in the position of maximum drag. For  $Re$ 's of 200 - 500 the motion was unpredictable.



Disks tended to wobble, while fuller bluff bodies tended to rotate.

Considering the results of this literature search it was apparent that this study initiated by Professor R. C. Hall of Kansas State University in 1955 was the first recognition of dynamic stability as a property of particles in an infinite expanse of fluid.

#### Illustration of Definition of Dynamic Stability

The following diagram, PLATE I, illustrates the two possible manners in which a disk could theoretically exhibit complete dynamic stability.

#### EXPLANATION OF PLATE I

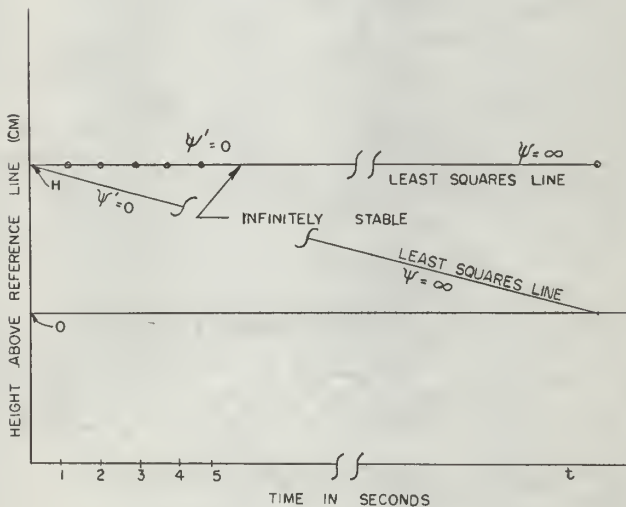
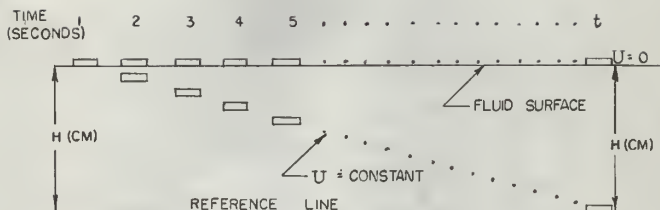
Illustration of infinitely stable motion of disks,

where  $\Psi' = \text{dynamic instability} = 0$ ,

$\Psi = \text{dynamic stability} = \infty$ , and

$U = \text{velocity}$ .

## PLATE I



## PROCEDURE OF MEASUREMENTS

### Outline of Procedure

For the study a series of 46 disks, approximately one inch in diameter and a quarter inch thick, were constructed. Disk 1 was lucite. Disks 2 through 26 had cores of stainless steel and rims of lucite. Disk 27 was all stainless steel. Disks 28 through 46 had lucite cores and stainless steel rims. By varying the size of the cores the disks were each given different weights and moments of inertia. Only part of the disks were used, due to the deterioration of the rest.

The procedure was to drop each disk in turn through a liquid, take motion pictures of the descent, and measure on a projection of each frame of the movie the disk's relative height above a bottom reference line (PLATE XII).

### Detailed Procedure

The procedure utilized in taking the movies of the disk's descent in a liquid media required the services of assistants A and B and photographer P (PLATES II, III, and IV). For the descent of each disk the procedure was as follows:

- 1.) As assistant B changed the cards designating the disk number, the drop number, and the liquid temperature, assistant A placed the designated disk on the particle vacuum release mechanism performing operations 1, 2, and 3 (PLATE V).
- 2.) B then turned on the foreground lights illuminating the

information cards. This signaled P to photograph this information.

- 3). Having built up, during a twenty second count, sufficient vacuum to hold the disk the line from the vacuum pump was closed and 'bled' by A in performing operations 4 and 5 (PLATE V). The release mechanism holding the disk was then lowered into the liquid.
- 4.) At the end of fifty-five seconds of a second count, A notified B to signal P by means of a buzzer to commence photographing. At sixty seconds the remaining vacuum was released by performance of operation 6 (PLATE V), thus allowing the disk to start its descent through the liquid.
- 5.) As the particle reached the bottom of the column, B signaled the photographer to cease operation.
- 6.) This procedure was repeated for successive disks.

An approximately identical method of release for the disks was obtained by means of the particle vacuum release mechanism. By allowing a time lapse, the twenty and sixty second counts, and by floating Agile plastic stars (See Table of Definitions, Appendix) on the fluid surface, the internal fluid agitation was minimized to a satisfactory degree.

Following the development of the film, measurements on the descent of the disk were started. For this operation assistants A and B were again utilized. The assistants were selected on the basis of the similarity between their measurements over several trials. This likeness in observation of the measured quantity allowed the assistants to be rotated between jobs within any set of measurements. To further

minimize fatigue and monotony a third assistant C was used when available.

The film was threaded through a special adaptor (PLATE XI) on the projector. The use of this adaptor largely prevented the film from shifting or buckling, as well as adapting the 16mm. film to a 35mm. projector. Prior to starting the measurements the projector was allowed to run twenty to thirty minutes, thus allowing distortion caused by heat from the projection lamp to reach near equilibrium in the film. The focus on the projector was earlier set at an optimum position; this was determined by repeated adjustments of the focus until both assistants were visually satisfied.

The actual measuring operation was in general performed as follows:

- 1.) After the projector was allowed to heat for twenty to thirty minutes before the day's run, the disk number, liquid temperature, and drop number were identified by assistant B from an identification frame (PLATE XII, Fig. 1) and called to assistant A to be recorded.
- 2.) The film was then pulled through the projector by assistant A until the frame was found showing the disk to be located just below the top reference line. This frame was the first to be measured for each disk descent.
- 3.) At this point necessary minor adjustments were made in the frame location on the screen. By means of the turnbuckles and the focus on the projector (PLATE VI and VII) the projected picture of the frame was re-

stricted between vertical boundary lines drawn on the screen (PLATE VIV). By pulling the film up or down the image of the top reference line was placed in approximate line with a reference mark drawn on the screen (PLATE VIV). These adjustments minimized any possible error in measurements. These errors occurred from minute differences between each frame's position in the projector due to the slight bending of the film and the minor deviations of the film from the vertical in passing each frame through the adapter (PLATE XI).

- 4.) After these preliminaries were finished the readings on a frame were started. By means of a target (PLATES VI, IX, and X), B located the bottom reference line, the particle, and the top reference line positions, respectively. These locations were in turn transmitted by a system of pulleys to a stationary measuring device (PLATE VI and VIII) where the values  $L_H$ ,  $L_P$ , and  $L_L$  were read and recorded by A. For reasons of uniformity in the tension on the pulley wire the target was always adjusted downward onto the image being measured. For statistical reasons the readings were repeated six times for each frame. For each reading, by means of a signal light, B notified A when the target was set and by means of a buzzer, A notified B when he had completed the reading (PLATE VIII).
- 5.) After completing this first frame a second frame was placed in position. For each frame, after the initial frame of a day's measurements, a ten to fifteen second time lapse was

allowed so that the frame had sufficient time to reach equilibrium with the distortions caused by the heat of the projection lamp striking the film.

- 6.) The same procedure was employed for each frame. The measurements for any one disk were terminated at the frame showing the particle in its final position above the bottom reference line.

By utilization of a true length correction factor for each frame,  $\frac{100}{L_p - L_L}$ , changes in the film due to moderate temperature and humidity changes were prevented from causing measuring errors. More rapid distortion due to extreme temperature changes in the film as caused by the heat from the projection lamp were minimized by short time lapses between each frame read in which equilibrium was reached in the film. Fatigue in the measuring personnel was minimized by changes in duties every half hour to one hour, depending on the weather and by taking frequent rest breaks.

Errors in measurement caused by shifts in the picture projection due to extraneous disturbances were avoided by the replacement of such data after the disturbance had ceased. Minor disturbances and vibrations were alleviated by the location of rubber padding under the projector platform and by the use of several restraining wires around the projector (PLATE VI and VIII).

#### Diagrams of Equipment

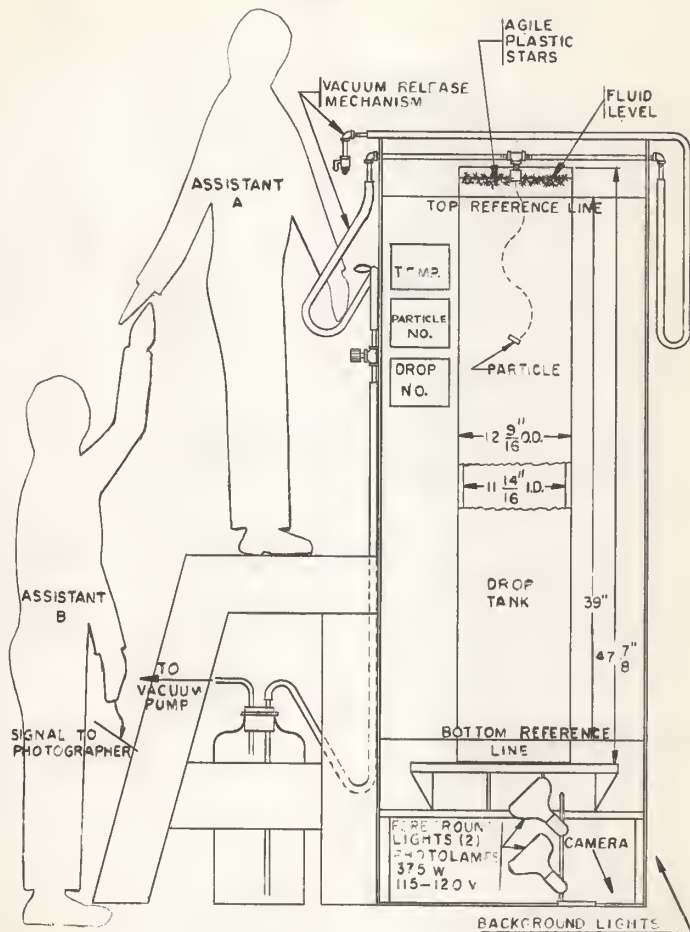
The following pages present diagrams of the experimental equipment and illustrate procedures.



#### EXPLANATION OF PLATE II

Front view of particle drop column showing  
lighting arrangement and personnel placement  
for photography of disk descent in liquid  
column.

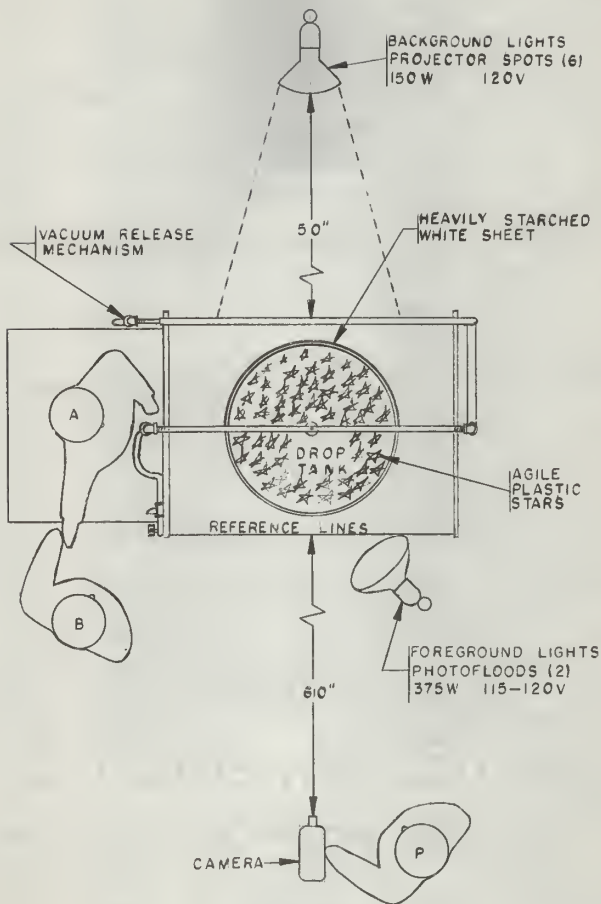
PLATE II



#### EXPLANATION OF PLATE III

Top view of particle drop column showing personnel,  
lighting and camera positions for photography of  
disk descent in fluid media.

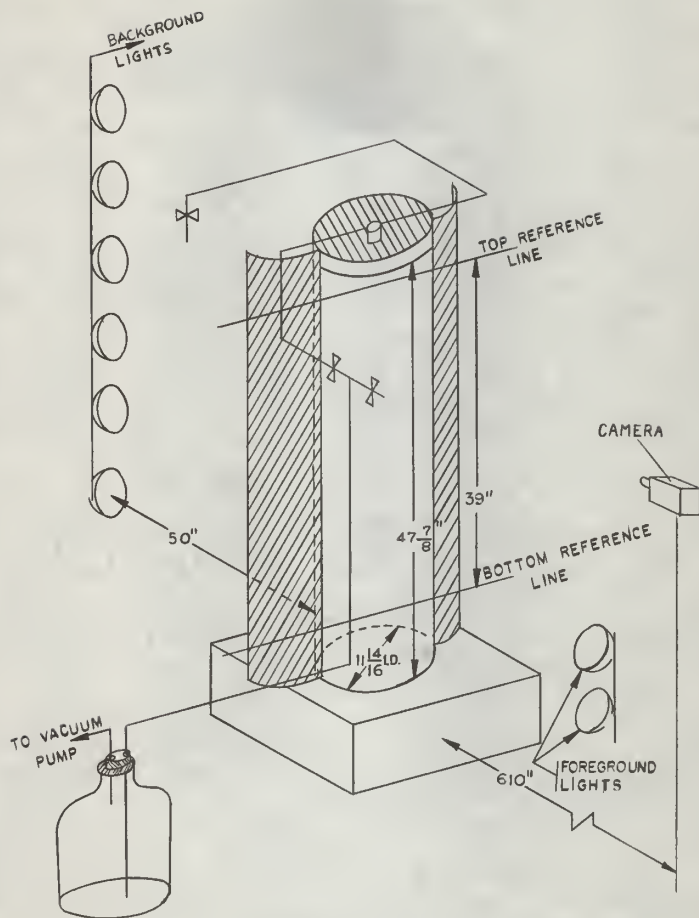
PLATE III



#### EXPLANATION OF PLATE IV

Schematic of arrangement for photography of  
disk descent in fluid media in glass column.  
///// heavily starched white sheet as back-  
ground of drop tank.

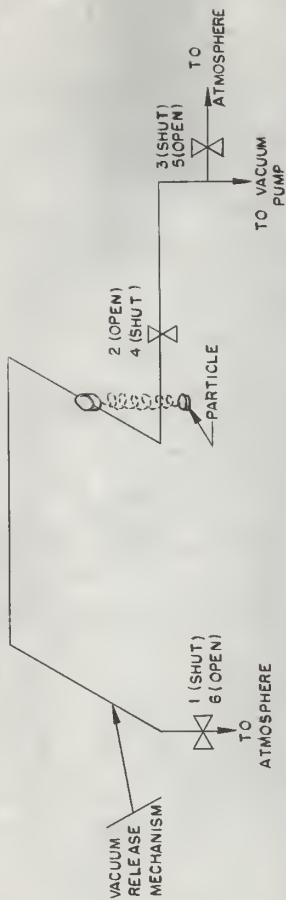
PLATE IV



#### EXPLANATION OF PLATE V

Schematic of particle vacuum drop mechanism  
giving order of operation of valves in actual  
release of disk in fluid media.

PLATE V



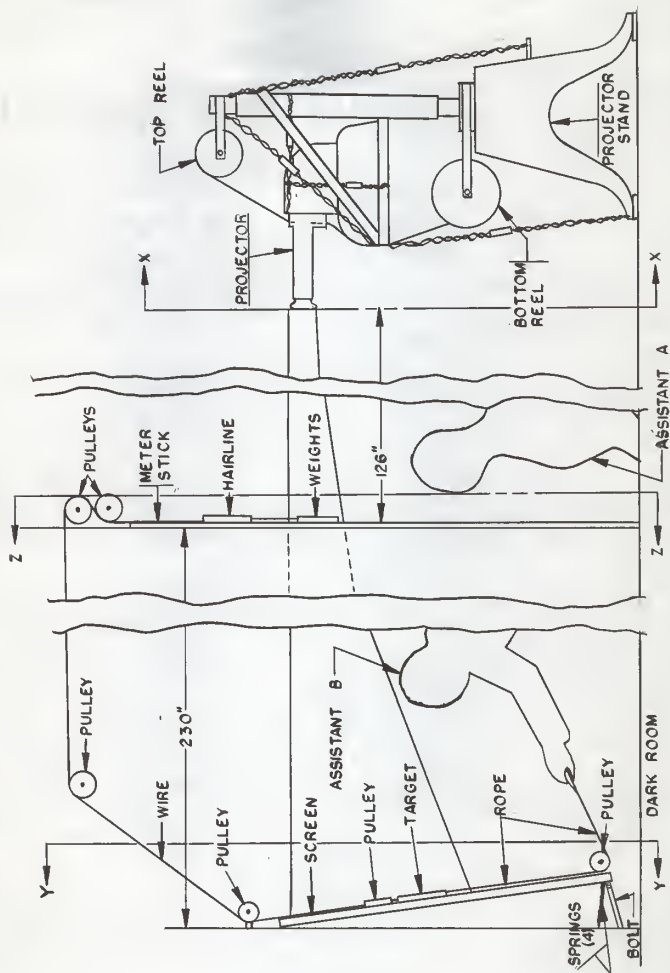


#### EXPLANATION OF PLATE VI

Side view of personnel and equipment for measuring  $L_L$ ,  $L_P$ , and  $L_H$  on film taken in PLATES II, III, IV, and V.

Shows placement of projector, reading device, dark room, and screen.

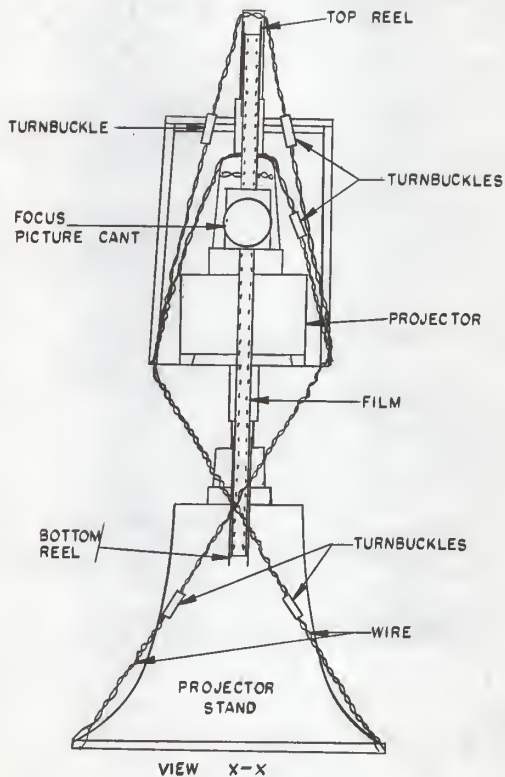
PLATE VI



#### EXPLANATION OF PLATE VII

Front view of projector showing location of turnbuckles for right and left adjustments of film projection and focus for refinement and right and left cant of projection.

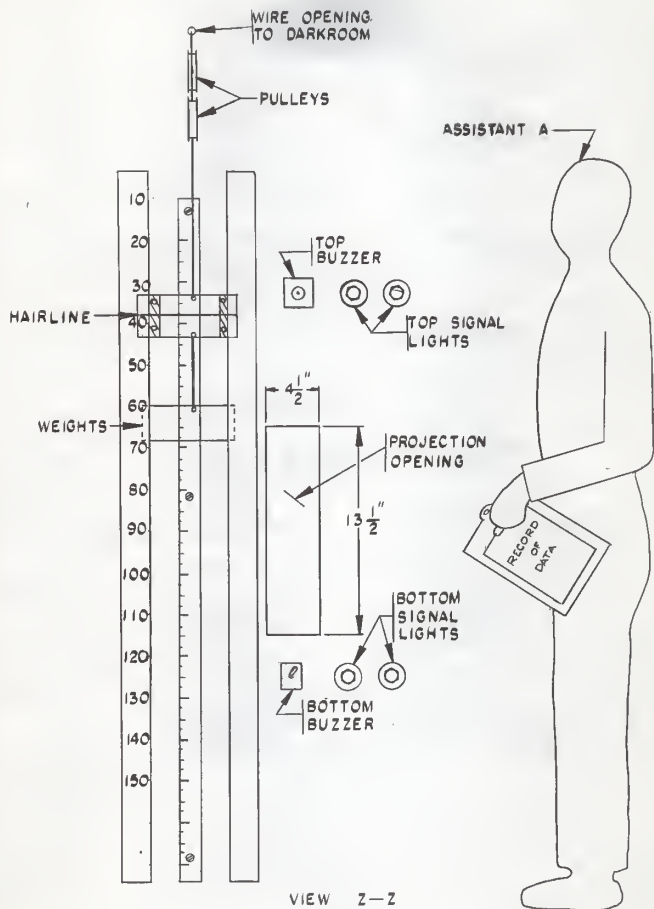
## PLATE VII



#### EXPLANATION OF PLATE VIII

Front view of measuring device showing  
assistant A, projectionist and reader-  
recorder of  $L_L$ ,  $L_P$ , and  $L_M$  values.

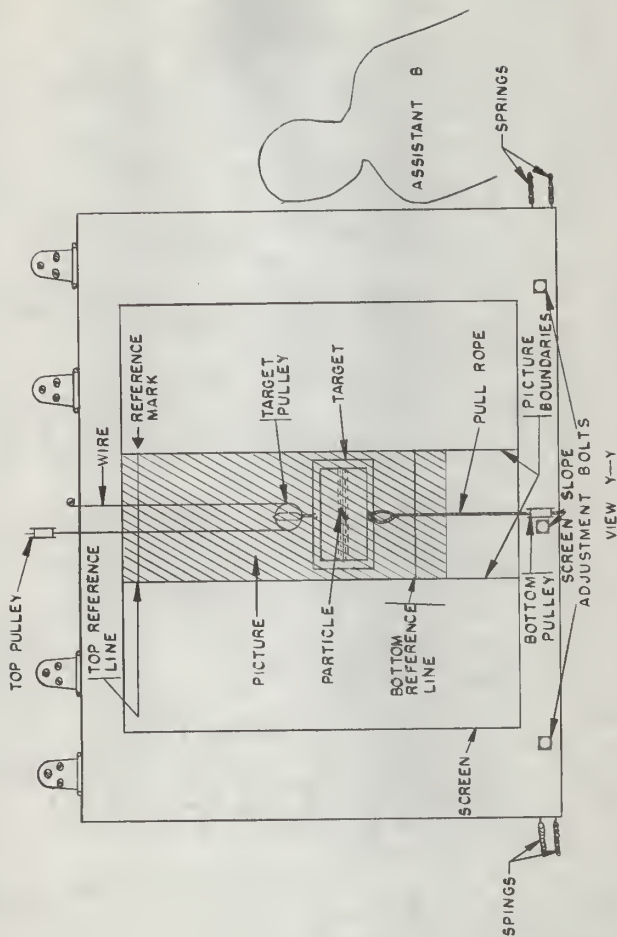
## PLATE VIII



#### EXPLANATION OF PLATE IX

Front view of screen showing location of picture, target, and assistant B - target adjuster.

PLATE II

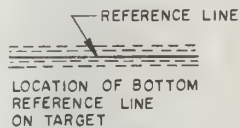
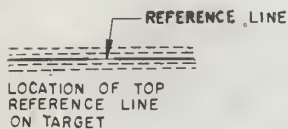
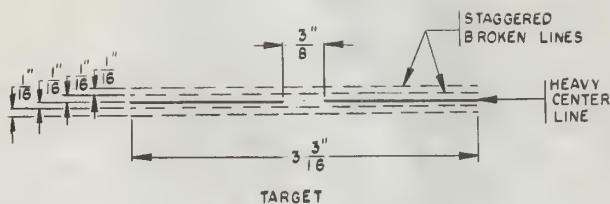




EXPLANATION OF PLATE X

Front view of target, showing actual application  
of target to three measured elements of each frame.

## PLATE X

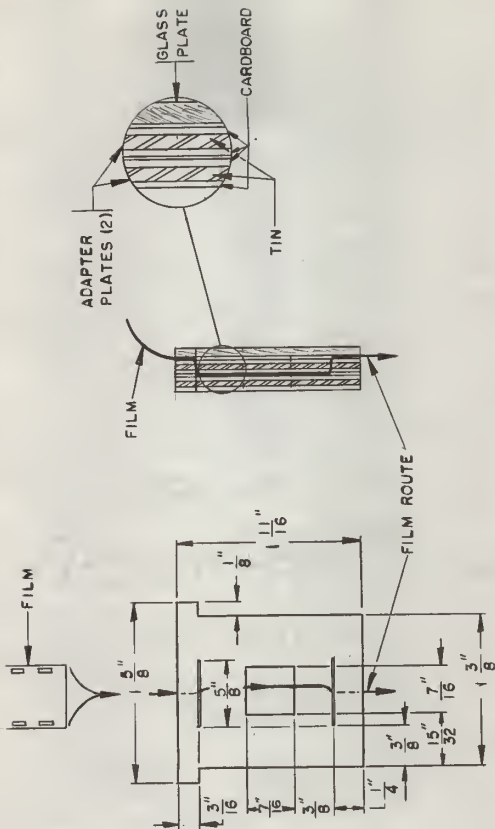


OPERATION OF TARGET

EXPLANATION OF PLATE XI

Front and side views of adapter  
of 16mm film to 35mm projector.

PLATE XI



ADAPTER PLATE

# EXPLANATION OF PLATE XII

Frames from movie of the descent of a disk.

Fig. 1. Identification frame.

/// temperature of fluid media

\\\\ disk number

≡ drop number

Fig. 2. Measuring frame.

$L_H$  = top reference line

$L_p$  = disk

$L_L$  = bottom reference line

## PLATE XII

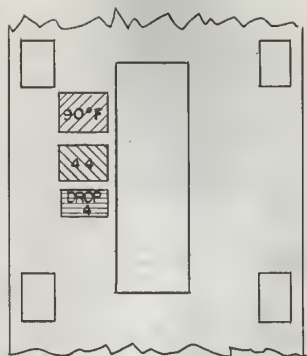


Fig. 1

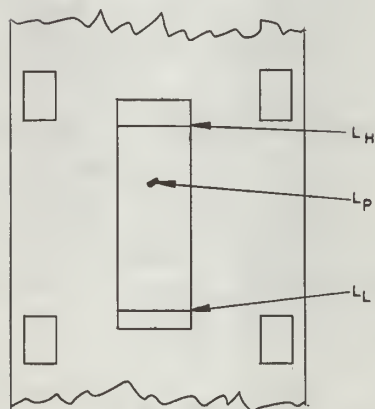


Fig. 2

## MATHEMATICAL DEFINITION OF DYNAMIC STABILITY

Initial Calculations on Data from Representative Frame of a Descent

DATE 7-17-59		PARTICLE NO. 23		
θ	L <sub>H</sub>	L <sub>P</sub>	L <sub>L</sub>	θ
1	1	105.23	104.52	25.52
	2	105.22	104.56	25.51
	3	105.21	104.52	25.49
	4	105.20	104.52	25.50
	5	105.18	104.48	25.49
	6	105.18	104.49	25.51
	Ave	105.20	104.52	25.50
	L <sub>H</sub> -L <sub>L</sub>	79.70	L <sub>P</sub> -L <sub>L</sub>	79.01
2	T.L.	1.25470	h <sub>i</sub>	99.14
	1	102.51	99.38	22.72
	2	102.51	99.32	22.71
	3	102.49	99.30	22.71
	4	102.49	99.34	22.71
	5	102.47	99.32	22.68
	6	102.50	99.31	22.68
	Ave	102.50	99.33	22.70
	L <sub>H</sub> -L <sub>L</sub>	79.80	L <sub>P</sub> -L <sub>L</sub>	76.43
	T.L.	1.25313	h <sub>i</sub>	96.03
	1	102.91	96.88	23.29
	2	102.89	96.90	23.30

θ = frame number.

L<sub>H</sub> = measured height of top reference line, relative to base line.L<sub>P</sub> = measured height of diek, relative to base line.L<sub>L</sub> = measured height of bottom reference line, relative to base line.
$$T.L. = \frac{100}{L_H - L_L} = \text{factor for adjusting relative values to true values.}$$

$$h_1 = (L_P - L_L) \left( \frac{100}{L_H - L_L} \right) = \text{true height of disk above bottom reference line.}$$

(See PLATE XIII, Fig. 1 for physical descriptions of  $L_H$ ,  $L_P$ , and  $L_L$ ).

For first reading on first frame:

$\theta$	Reading	$L_H$	$L_P$	$L_L$
1	1	105.23	104.52	25.52

$$L_H - L_L = 79.71$$

$$L_P - L_L = 79.00$$

$$T. L. = \frac{100}{L_H - L_L} = 1.25455$$

$$h_1 = (L_P - L_L) (T.L.) = 99.11$$

For readings 2, 3, 4, 5, 6 on frame one - above procedure repeated.

Then:

$$h_{i1} = 99.11$$

$$h_{i2} = 99.17$$

$$h_{i3} = 99.13$$

$$h_{i4} = 99.15$$

$$h_{i5} = 99.12$$

$$h_{i6} = 99.13$$

$$h_{i \text{ avg.}} = 99.14 = h_i^-$$

Calculation of 90% confidence interval on  $h_i^-$  of frame one:

$n$	$(h_{i_n} - h_i^-)$	$(h_{i_n} - h_i^-)^2$
1	-0.03	0.0009
2	+0.03	0.0009
3	-0.01	0.0001
4	+0.01	0.0001
5	-0.02	0.0004
6	+0.01	0.0001

$$\sum (h_{i_n} - h_i^-)^2 = 0.0025$$



$$s_1^2(h_1) = \frac{\sum (h_{1n} - h_1^-)^2}{n-1} = \frac{0.0025}{5} = 0.0005$$

$$s^2 = \frac{\sum (h_{1n} - h_1^-)^2}{n(n-1)} = \frac{0.0005}{6} = 0.000083$$

$$s = \sqrt{s^2} = \sqrt{0.000083} = 0.0091$$

$$t_{0.05} = \text{Student's } t \text{ for } 95\% \text{ confidence interval} = 2.571$$

$$(t_{0.05})(s) = 2.571(0.0091) = 0.0234$$

$$h_{1t_{0.05}} = 99.14 \pm 0.02$$

$$99.12 \leq \bar{h}_1 \leq 99.16$$

The confidence intervals on the measurements provided a means of detecting errors that occurred at any time during the processing of the data.

Least squares line (sample regression line):

Observation Number	$\theta$	$h_{1\theta}$	$\theta^2$	$\theta h_{1\theta}$
1	1	99.14	1	99.14
2	2	96.03	4	192.06
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.
$\theta$	.	.	.	.
	$\sum \theta$	$\sum h_{1\theta}$	$\sum \theta^2$	$\sum \theta h_{1\theta}$

$$(\text{Number of observations}) k + (\sum \theta) j = \sum h_{1\theta}$$

$$(\sum \theta) k + (\sum \theta^2) j = \sum \theta h_{1\theta}$$

Solve simultaneous equations for  $k$  and  $j$ .

Equation of least squares line:

$$y = k + jx$$

$$k = y \text{ intercept at } \theta = 0$$

$j$  = slope of least squares line, or  
sample regression coefficient

Definition of Dynamic Stability,  $\psi_{s^2_{y.x}}$  and  $\psi_{\sqrt{s^2_{y.x}}}$

At any frame,  $\theta$ , the absolute deviation from regression of the height of a disk from the height of the least squares line through its path =  $h_{1\theta} - (k + \theta j)$ , where

$h_{1\theta}$  = the height of the disk at  $\theta$ ,

and  $k + \theta j$  = the height of the least squares line at  $\theta$ .

Assumption: Each frame over the descent of a disk was equal to the same increment of universal time, where the conversion factor of frames to seconds was  $F$ .

For any randomly selected descent of a disk consisting of  $O$  frames, the summation of the squares of absolute deviation from regression =

$$\begin{aligned} & \left| h_{1_1} - (k + j) \right|^2 \text{ times (number of frames having this} \\ & \quad \text{absolute deviation, which was always one) times} \\ & (F) + \left| h_{1_2} - (k + 2j) \right|^2 (1) (F) + \dots \\ & \dots + \left| h_{1_\theta} - (k + \theta j) \right|^2 (1) (F). \end{aligned}$$

Units:

$$\left| h_{1_1} - (k + \theta j) \right|^2 = \text{cm}^2.$$

number of frames having above absolute deviation = frames.

$$F = \frac{\text{seconds}}{\text{frames}}$$

$$\text{cm}^2 \frac{\text{frame}}{\text{seconds}} = \text{cm}^2 \frac{\text{seconds}}{\text{frames}}$$

Thersfors:

Summation of absoluts daviations from regression =

$$\left| h_{i\theta} - (k + \theta j) \right|^2 (1)(F) = F \sum |d_{y.x}|^2.$$

Finally:

$$\begin{aligned} \psi'^2_{s^2_{y.x}} &= F \sum |d_{y.x}|^2 / (\theta - 2)(F) \\ &= \text{msan square deviation from regrssion, or} \\ \text{dynamic unstability} &= \frac{\text{cm}^2 \frac{\text{seconds}}{\text{frames}}}{\frac{\text{seconds}}{\text{frames}}} \\ &= \text{cm}^2. \end{aligned}$$

$\theta - 2$  = total number of frames minus two, since  
two degrees of freedom were lost in the  
calculation.

$$\text{Dynamic stability} = \psi'^2_{s^2_{y.x}} = \frac{1}{\psi'^2_{s^2_{y.x}}} = \frac{1}{\text{cm}^2}$$

$$\text{Or, dynamic stability} = \frac{\psi'^2_{s^2_{y.x}}}{\psi'^2_{s^2_{y.x}}} = \frac{1}{\text{cm}^2}$$

Definition of Dynamic Stability,  $\psi_{\text{AREA}}$

(Sse PLATE XIV, Fig. 1, whsrs the shaded area on this plot represents

$$\psi'_{\text{Area}_{\theta}}.)$$

Trapszoidal Ruls:

(Sss PLATE XIV, Fig. 2 and 3)

$$\begin{aligned} ab &= \left| h_{i\theta} - (k + \theta j) \right| \\ cd &= \left| h_{i\theta+1} - [k + (\theta + 1)j] \right| \\ ef &= \left| h_{i\theta+2} - [k + (\theta + 2)j] \right| \\ gh &= \left| h_{i\theta+3} - [k + (\theta + 3)j] \right| \end{aligned}$$

On Fig. 3:

$$\frac{ab + cd}{2} = wy = xz$$

$$\text{Area } wyzx = \text{area } abdc$$

$$yz = wx = (1) F =$$

$$\text{frames} \frac{\text{seconds}}{\text{frames}} = \text{seconds}$$

$$\text{Area abdfhgeca} = \psi' \text{Area} = \frac{ab + cd}{2} (1)(F) + \frac{cd + ef}{2} (1)(F) + \frac{ef + gh}{2} (1)(F) = \left( \frac{ab}{2} + cd + ef + \frac{gh}{2} \right) F.$$

Where the experimental curve crosses the least squares curve between consecutive positions of the disk a value C must be calculated. (See PLATE XIV, Fig. 4)

$$\text{Area a'b'c'd'a'} = \frac{\left\{ \left| h_{i\theta} - (k + \theta j) \right|^2 + \left| h_{i\theta+1} - [k + (\theta + 1)j] \right|^2 \right\}}{2 \left\{ \left| h_{i\theta} - (k + \theta j) \right| + \left| h_{i\theta+1} - [k + (\theta + 1)j] \right| \right\}} F = C$$

Situation 1. (See PLATE XIII, Fig. 2)

$$\text{Area} = \text{Area a12345678b}$$

$$= \left\{ \frac{\left| h_{i1} - (k + j) \right|}{2} + \frac{\left| h_{i2} - (k + 2j) \right|}{2} + \frac{\left| h_{i3} - (k + 3j) \right|}{2} + \frac{\left| h_{i4} - (k + 4j) \right|}{2} + \frac{\left| h_{i5} - (k + 5j) \right|}{2} + \frac{\left| h_{i6} - (k + 6j) \right|}{2} + \frac{\left| h_{i7} - (k + 7j) \right|}{2} + \frac{\left| h_{i8} - (k + 8j) \right|}{2} \right\} (F)$$

Situation 2. (See PLATE XIII, Fig. 3)

$$\text{Area} = \text{Area a12345678b}$$

$$= \left\{ \frac{\left| h_{i1} - (k + j) \right|}{2} + \frac{\left| h_{i2} - (k + 2j) \right|}{2} + \frac{\left| h_{i3} - (k + 3j) \right|}{2} + \frac{\left| h_{i4} - (k + 4j) \right|}{2} + \frac{\left| h_{i5} - (k + 5j) \right|}{2} + \frac{\left| h_{i6} - (k + 6j) \right|}{2} + \frac{\left| h_{i7} - (k + 7j) \right|}{2} + \frac{\left| h_{i8} - (k + 8j) \right|}{2} \right\} F$$

Situation 3. (See PLATE XIII, Fig. 4)

$$\text{Area} = \text{Area al23456789b}$$

$$= \left\{ \frac{|h_{i_1} - (k + j)|}{2} + |h_{i_2} - (k + 2j)| + |h_{i_3} - (k + 3j)| \right. \\ + |h_{i_4} - (k + 4j)| + |h_{i_5} - (k + 5j)| + |h_{i_6} - (k + 6j)| \\ \left. + |h_{i_7} - (k + 7j)| + |h_{i_8} - (k + 8j)| + \frac{|h_{i_9} - (k + 9j)|}{2} \right\}_{(F)},$$

where terms 4 and 7 are zero.

In summary:

$$\text{Area} \frac{\text{AREA}_\theta}{(\theta-2) F} = \frac{(\text{cm.}) (\cancel{\text{frames}}) \frac{\cancel{\text{seconds}}}{\text{frame}}}{(\cancel{\text{frames}}) \frac{\cancel{\text{seconds}}}{\text{frame}}} = \text{cm.},$$

where F cancels the F in  $\psi'_{\text{AREA}_\theta}$

$$\psi_{\text{AREA}} = \text{Dynamic stability} = \frac{1}{\psi'_{\text{AREA}}} = \frac{1}{\text{cm.}}$$

#### Diagrams of Dynamic Stability

On the following pages diagrams are presented illustrating the methods of calculating dynamic stability.

# EXPLANATION OF PLATE XIII

## Definition of Dynamic Unstability.

Fig. 1. Physical significance of  $L_H$ ,  $L_P$ , and  $L_L$ ,  
on single frame of movie of disk descent.

Fig. 2. }  
Fig. 3. } Three possible situations in calculation  
Fig. 4. } of dynamic unstability,  $\Psi$  AREA<sub>g</sub>.

## PLATE XIII

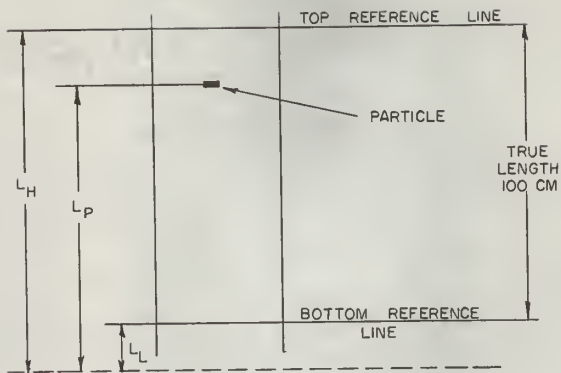


Fig 1

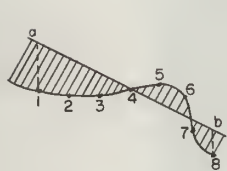


Fig. 2

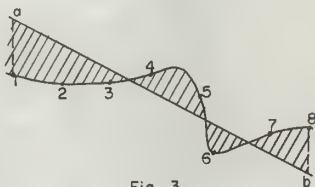


Fig. 3

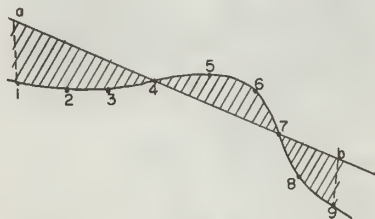


Fig. 4

#### EXPLANATION OF PLATE XIV

Definition of Dynamic Unstability,  $\Psi'_{\text{AREA}_0}$

Fig. 1. Illustration of  $\Psi'_{\text{AREA}_0}$ , where shaded area adjacent to least squares line represents  $\Psi'_{\text{AREA}_0}$ .

Fig. 2. } Illustration of application of trapezoid rule to  
Fig. 3. } calculating area,  $\Psi'_{\text{AREA}_0}$ .

Fig. 4. Illustration of calculation of area,  $\Psi'_{\text{AREA}_0}$ , in special case where trapezoid rule is not applicable.



## PLATE XIV

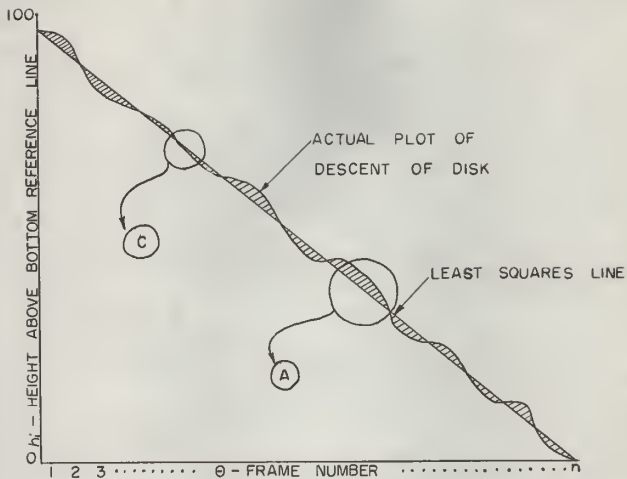


Fig. 1

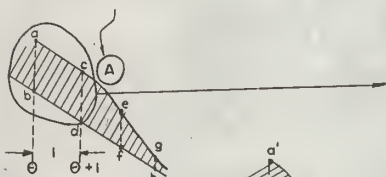


Fig. 2

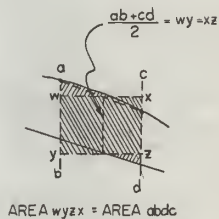


Fig. 3

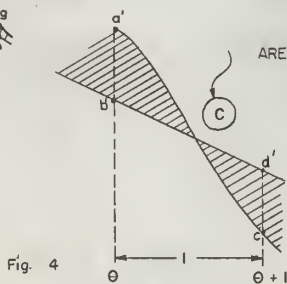


Fig. 4

## GENERAL VISUAL OBSERVATIONS ON THE DESCENT OF DISKS THROUGH TRIETHYLENE GLYCOL AND WATER

### Discussion of Observations and Postulations

Repeated observations were made of the general behavior of the disks as each descended through two separate fluid media; triethylene glycol and water. For water seventeen disks were investigated, while for triethylene glycol sixteen were investigated due to the buoyancy of disk one in the glycol. Preliminary to these investigations both faces of each disk were marked with a heavy black line corresponding to the diameter of the disk. In dropping these disks through either triethylene glycol or water the lines were visible to an observer looking down through the liquid column from a position above the drop tank. From this position any movement of the diameter of the disk in a plane parallel to the surface of the earth could be observed.

The disks were dropped in the fluid media at temperatures of  $80^{\circ} \pm 3^{\circ}\text{F}$  for water and  $90^{\circ} \pm 3^{\circ}\text{F}$  for triethylene glycol. At  $80^{\circ}\text{F}$  the density of water was 0.9963 and at  $90^{\circ}\text{F}$  that of one hundred per cent triethylene glycol was 1.115. However, at  $90^{\circ}\text{F}$  the viscosity of triethylene glycol was 22 centipoises while at  $80^{\circ}\text{F}$  that of water was approximately 0.037 times this or 0.82 centipoises.

Observations of the descents of each disk in triethylene glycol showed that the marked diameter normally did not rotate throughout the descent. With five observations of each disk this was observed in 96 per cent of all the observations. Of the disks in triethylene glycol which exhibited rotation of the marked diameter, this rotation appeared in only one out of

five observations made on each of these disks. In all cases this rotation was observed to be less than  $15^{\circ}$ . In triethylene glycol the disk occasionally exhibited rotational motion of the diametrical axis of maximum slope, but even in the majority of these cases the marked diameter remained aligned with the direction which it assumed when released (See Fig. 2, PLATE XVII). These observations were tabulated in Table 10, Appendix.

In water, the less viscous media, the marked diameter showed decided deviations from the direction it assumed when initially released for 65 per cent of the one hundred observations. In water at  $80^{\circ} \pm 3^{\circ}\text{F}$  two of the heavier disks, 30 and 32, exhibited no diameter rotation over five observations of their descents. The remaining fifteen disks exhibited rotation of the marked diameter in from one to all of the observed drops. In the case of disks 19, 23, 40, 43, and 45, rotations first in one direction and then back in the other were noted in part of the observations. Also it was observed, by the utilization of a second observer watching perpendicular to the descent of the disk, that many of the rotations occurred after the disk had descended three-fourths of the depth of the column. These observations were tabulated in Table 11, Appendix.

Other observations were made on the path of motion which the center of the disk followed as it descended. When viewed from above the liquid column these observations showed that in ninety-seven per cent of five descents each of sixteen different disks the path of a given disk remained within one vertical plane. A sighting mechanism, built of welding bars, was constructed in the shape of a circle, the same size as the top of the drop column, and with a single bar as a diameter of the circle. The apparatus was fitted on the top of the drop column such that as a

disk descended it was possible to rotate the mechanism so that the diametrical bar was contained in the vertical plane through the path of motion of the descending disk. A pointer was added to the mechanism such that it pointed to values of angle inscribed along the rim of the drop column. The angle or angles between the vertical plane containing the disk path of motion and a vertical reference plane passing through  $0^\circ$  on the rim of the column were determined for four separate drops in triethylene glycol. These observations in Table 12, Appendix, showed that only 2 descents out of 64 exhibited a shift in the vertical plane passing through the path of motion of the disk through its 100 cm. descent.

No such conclusive observations were made for the seventeen disks descending in water. In water 57 per cent of the descents showed no single planar restriction of the path of motion as observed from the top of the column. Of the 43 per cent of the descents showing single planar motion 73 per cent of these observations were made on disks 20, 23, 27, 30, 32, 35, 37, and 40. In 32 per cent of the single planar motions the disk hit the side of the tank before the full 100 cm. between reference lines had been traveled. In no case did any disk show single planar wave, flutter, or oscillatory motion for all five observations in water at  $80^\circ \pm 3^\circ\text{F}$ . These observations were tabulated in Table 14, Appendix.

The observations in this study were restricted to disks descending in triethylene glycol and in water at Reynolds numbers above 78. After a multitude of observations, including those presented in Tables 13 and 14 of the Appendix, it was postulated that the disks descended in one or more of four main types of motion. These motions were named stable, flutter, wave, and spiral.

In stable motion the center of the disk fell along a vertical line. The vertical line was always perpendicular to the face of the disk. In triethylene glycol particles 6 and 46 descended in a stable manner in five out of five descents of each disk. In water no disk exhibited stable descent in any of the five drops tabulated in Table 14, Appendix. At least ten other drops not tabulated substantiated these observations over the same range of disks.

In flutter the disk exhibited what appeared to be a wobble or "teeter-totter" motion about a given fulcrum. This fulcrum consisted of one particular diameter of the disk; this diameter did not change on the disk throughout the descent. Flutter occurred while the center of the disk was descending through either a vertical line or a three dimensional curvilinear path. In triethylene glycol this diametrical fulcrum remained constant in direction throughout the descent.

In triethylene glycol disks 9, 13, 20, and 45 exhibited what was observed as flutter in one, two, or three descents out of five. Disks 19, 20, and 30 exhibited flutter as a component of wave motion. In water disks 13, 16, 19 and 20 appeared to descend with a flutter in at least one observation out of seven. In Table 14, Appendix, only five of these observations were tabulated. It was found in the case of flutter that a second or third observer sometimes viewed the motion as a small wave, instead of flutter. This led to later decisions concerning flutter as a postulated type of motion.

Wave motion was described as an oscillation of the center of the disk with the face of the disk always remaining perpendicular to the direction of motion (See Fig. 3, PLATE XV). In oscillation the path

of motion of the center of the particle, when projected onto a vertical plane, appeared similar to a sine or cosine curve. As already described for descents in triethylene glycol a single vertical plane contained the entire path of oscillatory motion. For water this restriction to a single vertical plane did not apply, as oscillation and other motions in water were unmistakably three dimensional. In triethylene glycol 75 per cent of the 80 observations in Table 13, Appendix, revealed wave motion alone or in combination with flutter or disk rotation without diameter rotation. In water wave motion alone, or in combination with spiral or disk rotation with diameter rotation, appeared in 67 per cent of the 85 observations of 3 drops in Table 14, Appendix. Spiral was defined as a "cork screw" path of motion of the center of the disk (See Fig. 1, PLATE XVII). In Table 13, Appendix, spiral motion did not appear in 5 drops through triethylene glycol. Other observations made in this study, but not tabulated, substantiated this. In water disk one exhibited exclusively spiral motion over three drops as shown in Table 14, Appendix. In 29 per cent of the total observations of seventeen disks in Table 14, Appendix, spiral motion appeared either alone or as an extension of wave motion and flutter. The two dimensional appearances of these motions were illustrated in PLATES XV and XVII.

Other postulated types of motion were also recognized but these motions only seldom or never appeared in triethylene glycol. The nomenclature given two of these motions were disk rotation without diameter rotation and disk rotation with diameter rotation. In both cases disk rotation was basically defined as rotation of the diametrical axis of maximum slope of a disk. In the former case the disk did not actually

rotates, but, without a diameter reference mark, appeared to do so (See Fig. 2, PLATE XVII). In the latter case the disk rotated as apparent from the rotation of a marked diameter on the face of the disk (See Fig. 3, PLATE XVII). It was found that some discernment was necessary in distinguishing between borderline cases of spiral, disk rotation, and wave.

If the motion, appearing as a wave over a small increment of the total descent, exhibited at the same time one complete loop of a spiral over the total descent it was then classified as spiral instead of wave. If the motion appearing as a disk rotation exhibited any recognizable segment of a spiral loop over the total descent, it was classified as a spiral. In the case of disk rotation and wave motion together, the combination was treated as wave motion with rotation. Infrequently, in only 5 per cent of the observations in Table 13, Appendix, disk rotation without diameter rotation appeared in triethylene glycol, while disk rotation with diameter rotation did not appear in any of the 80 observations. In water, disk rotation with diameter rotation appeared in less than 5 per cent of the observations in Table 14, Appendix, while disk rotation without diameter rotation was not apparent in any of the 85 observations. Diameter rotation in water was most closely associated with spiral motion as was seen from a comparison of Tables 11 and 14, Appendix. These were illustrated in PLATE XVII.

At this point it was discovered that fairly accurate reproductions of the disk path of descent could be obtained. This was accomplished by tracing with a pencil the projected image of the disk from each movie frame, as each frame in turn, was projected onto a screen of white paper. Prior to tracing, each frame was adjusted to the position on the screen which corresponded to that previously occupied by the



preceding frame traced. This positioning was accomplished by adjusting the upper reference line pictured on each frame of the movie to a constant reference point on the white paper background; by this technique a fairly accurate reproduction of the overall path of descent was obtained. Only parts of the six descents filmed were analyzed in this manner due to the excess time and personnel requirements.

From these sketches more exacting pictures of the postulated types of motions of disks in triethylene glycol were established. The results of analysis of these sketches were tabulated in Table 15, Appendix. It was found that flutter was merely wave motion with barely perceptible horizontal displacement when viewed at the true velocity of descent. This observation was further substantiated by analyzing the remainder of the movies. For this analysis a vertical reference line was established by holding a white string vertically downward from the initial projected position of a disk as viewed on the movie screen. The projection of the disk was carefully observed as it descended along the string on the screen. Horizontal displacements to the right and left of this line immediately showed up on all the disks which had been formerly depicted as descending with a fluttering motion. These same operations, also, showed that the disks depicted as settling in stable motion did indeed settle in that manner. Typical results of these latter analyses were also tabulated in Table 15, Appendix.

From these newer approaches to observing motion it was postulated that wave motion actually consisted of three types. That previously classified as wave motion was reclassified as two types of motion - wave I and wave III. Wave I was subtyped as wave I-A and wave I-B.



Wave, as previously defined, became wave I-A. It was noted that wave I-A consisted of planar oscillatory motion and it was postulated that the disk always maintained orientation such as to offer the face of greatest resistance perpendicular to the path of motion of the center of the disk. Wave I-B motion was observed as similar in definition to wave I-A motion, but without the restriction that the face of greatest resistance always be perpendicular to the path of the center of the disk. Analysis of sketches of disk descents showed that as a disk became heavier its increased moment of inertia prevented it from behaving as a wave I-A disk. It was impossible to discern between these two types of motion on the sketches of the disk descent. However, as the moment of inertia of the disks increased, wave I-A and wave I-B motions formed a logical sequence between stable and wave III motions. Wave I-B motion was definitely observed in disks 19, 20, 23, and 27. However, for disks 9, 13, and 16 there was little to discern between the types of motion, since the sketches were not refined enough to allow precision study. The heavier a disk became the larger the absolute difference of  $\theta - 90^\circ$  became.  $\theta$  represented, at any instant, the smallest angle between the plane of the leading face of the disk and the directional vector of the center of the disk. Disks exhibiting wave I-A and I-B motion, at least once, were 16, 19, 20, 23, 27, and 30. These motions are illustrated in PLATES XV, XVI, and XVII.

When the plane containing the face of a disk rotated away from the horizontal through the vertical and back to the horizontal wave III motion was observed. It was postulated that the angle between the plane containing the face of the disk and a horizontal plane became increasingly

larger as the disk descended until the disk turned over or flipped. Flip was defined as the motion of a disk where the lower face of the disk interchanged with the upper face through a  $180^\circ$  rotation. Disks exhibiting wave III motion were disks with large moments of inertia with respect to their weights. In all cases they were of steel rim and plastic core construction. Therefore this flipping action was a result of the large moments of inertia of these disks. This was a reasonable conclusion when it was considered that a baton is constructed of a shaft with heavy weights at each end such that it has a large moment of inertia.

The motions described here and illustrated in PLATE XVI can be observed by the use of a sheet of typing paper. A rectangle 4 by  $1\frac{1}{2}$  inches cut from the sheet and dropped face down from shoulder high exhibits wave III motion. A square consisting of three  $2\frac{1}{2}$  by  $2\frac{1}{2}$  pieces glued together exhibits either wave I-A or wave I-B motions.

Further observations concerned the frequencies of planar oscillation of disks descending in wave patterns. From the sketches prepared for Table 15, Appendix, the following frequencies in cycles per 100 cm. descent were observed for parts of two different drops.

Table 1: Frequencies in Cycles per 100 cm. Descents of Disks.

Disk Number	Drop One	Drop Two
9	---	6
13	---	6
16	6	6
19	6	$5\frac{1}{2}$
20	---	$5\frac{3}{5}$
23	5	---
27	$4\frac{3}{4}$	---
30	4	---
32	4	---
35	---	---
37	$3\frac{2}{3}$	---
40	---	---

### Diagrams of Postulated Types of Motion

On the following pages diagrams are presented illustrating the postulated types of motion of disks descending in triethylene glycol and water.

EXPLANATION OF PLATE XV

Postulated types of motion of disks descending  
in a fluid media.

Fig. 1. Stable descent.

Fig. 2. Flutter.

Fig. 3. Wave I-A, one of two possible subtypes  
of Wave I.

PLATE XV

STABLE DESCENT



FLUTTER



WAVE 1-A

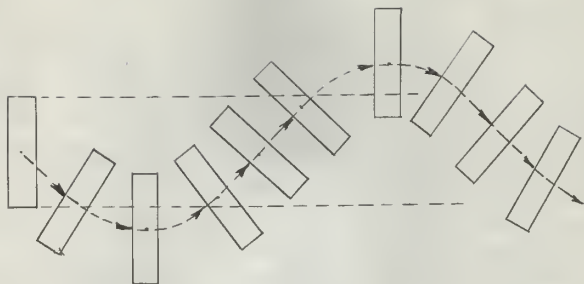


Fig. 1

Fig. 2

Fig. 3

#### EXPLANATION OF PLATE XVI

Postulated types of motion of disks descending in a fluid media.

Fig. 1. Wave I-B, the other subtypes of Wave I motion.

Fig. 2. Wave III.

## PLATE XVI

WAVE I-B

WAVE III

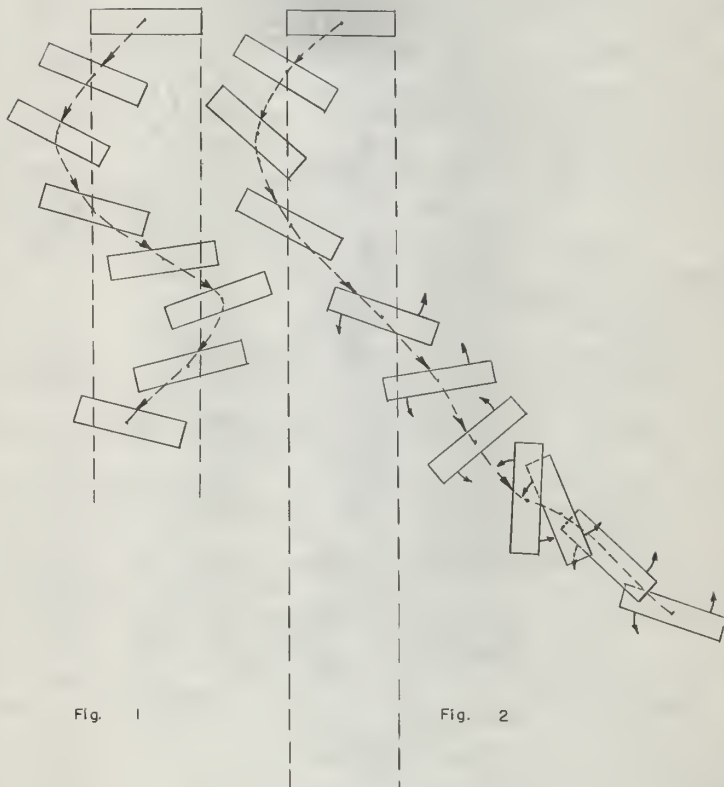


Fig. 1

Fig. 2

### EXPLANATION OF PLATE XVII

Postulated types of motion of disks descending in fluid media.

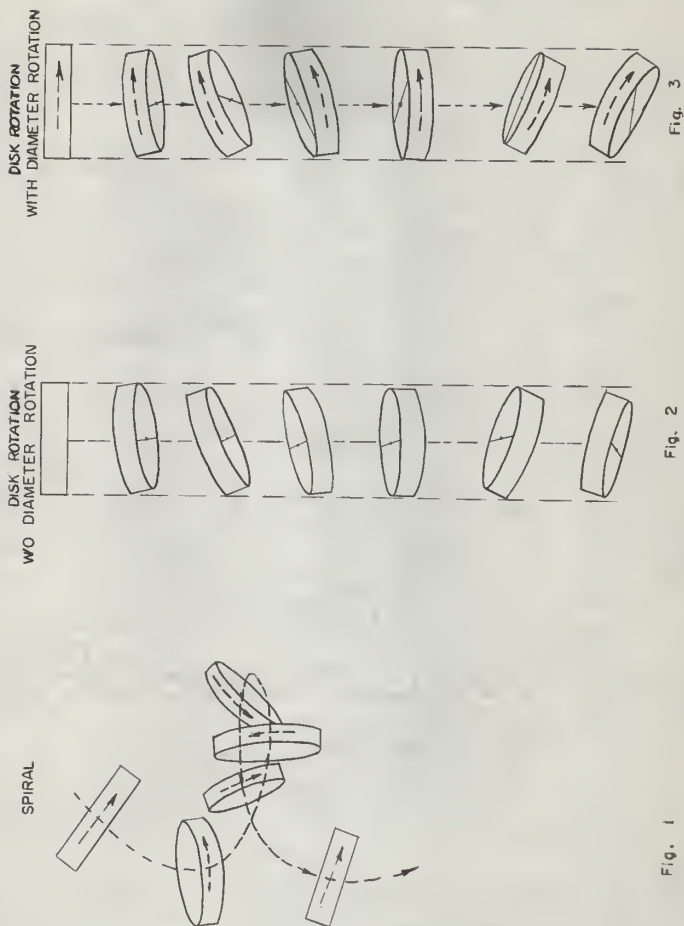
Fig. 1. Spiral.

Fig. 2. Disk rotation without diameter rotation.

Fig. 3. Disk rotation with diameter rotation.



## PLATE XVII



EXPLANATION OF PLATE XVIII

Illustrations of ascent of air bubble and descent of flat solid.

Fig. 1. Ascent of air bubble.

This illustration was acquired from Fig. 12 of Reference 5.

Fig. 2. The track of a flat solid.

This illustration was acquired from Reference 9.



## EVALUATION OF DATA AND PRESENTATION OF RESULTS


Dynamic stabilities were calculated by three methods; each method was previously described. The dynamic stabilities evaluated were  $\Psi_{s^2_{y,x}}$ , reciprocal of mean square deviation from regression,  $\Psi_{s^2_{y,x}}$ , reciprocal of sample standard deviation from regression, and  $\Psi_{AREA}$ , reciprocal of mean area between least squares line and path of disk descent. These values for three drops were tabulated in Tables 7, 8, and 9 of the Appendix. Obvious differences existed between the dynamic stabilities calculated by any one method for each disk over its three descents. The more outstanding reasons for these differences were the randomness of free-fall descents of disks in fluids, the paralactic errors, the measuring errors, and the extraneous disturbances, as described in the procedure, in the fluid media during descents of the disks.

It was impossible to distinguish between true randomness of motion and erraticness of motion which was induced by extraneous disturbances. Extraneously induced disturbances in the fluid were assumed to be sufficiently dampened by the precautions earlier described in the procedure, although these steps certainly guaranteed no possibility for complete removal of this source of error. Measuring errors, a result of obtaining values of disk height from motion pictures of disk descents, provided measureable but unpredictable sources of disagreement, dictated by equipment, personal, and environmental deficiencies at any time. Parallax - as treated by Himes (3) - also afforded measureable sources of error, where the magnitude of paralactic error was determined, for the

# EXPLANATION OF PLATE XIX

Illustration of 90% confidence intervals

on  $\psi \sqrt{s^2_{y.x}}$ , dynamic stability, for range  
of disks investigated.

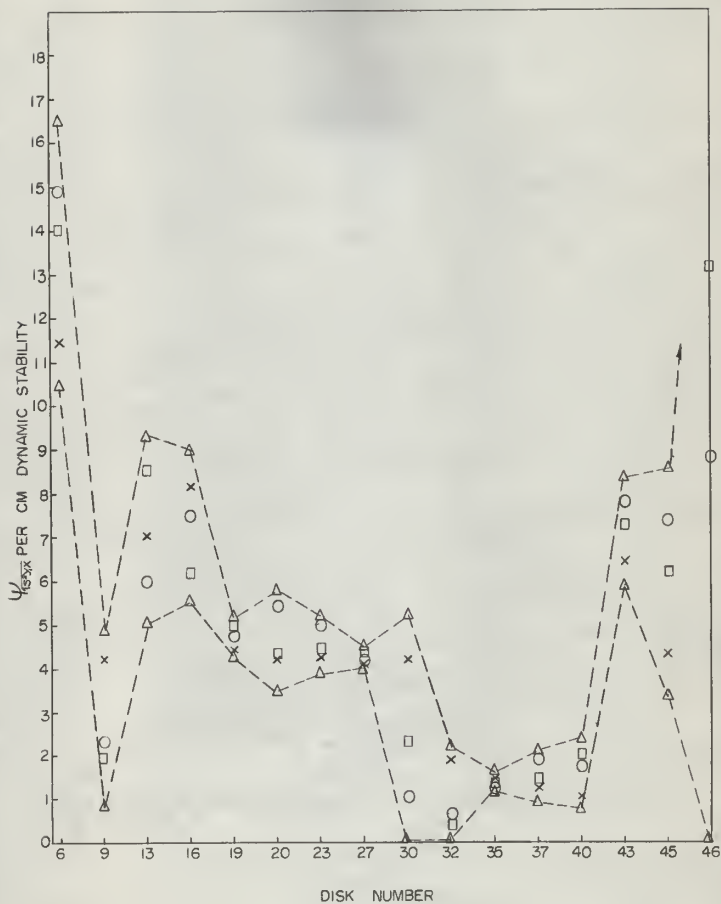
 upper and lower  
limits of 90%  
confidence intervals.

X Drop 1.

O Drop 2.

□ Drop 3.

PLATE XIX



# EXPLANATION OF PLATE XX

Plots of  $\Psi\sqrt{s^2_{y.x}}$ , dynamic stability, for three drops over entire range of disks showing intensities of  $\Psi\sqrt{s^2_{y.x}}$  in relation to postulated types of motion.

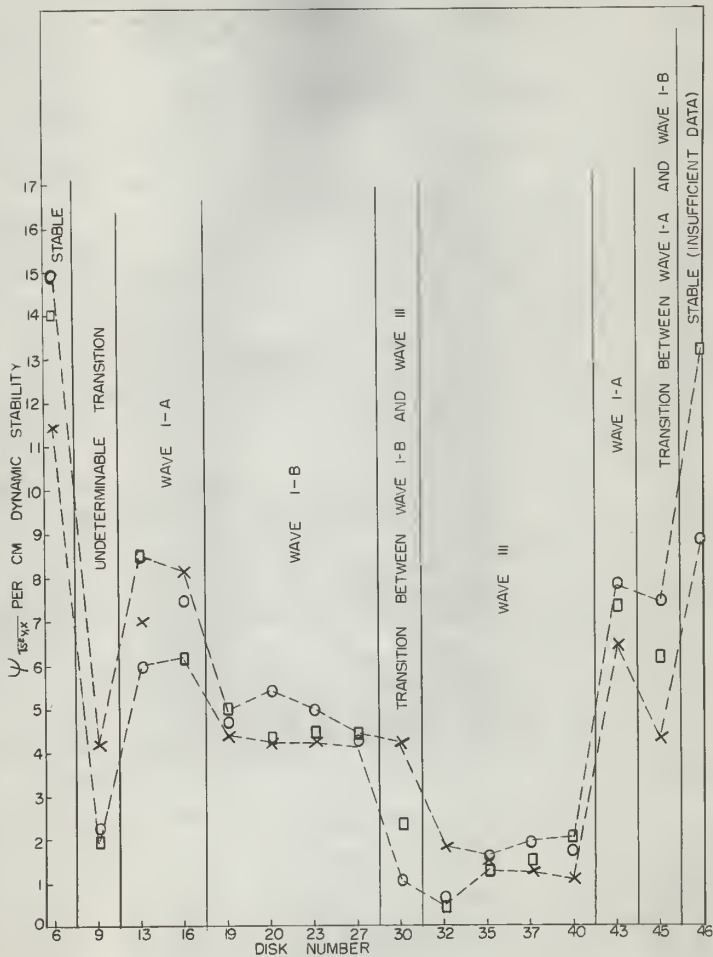
X Drop 1.

O Drop 2.

□ Drop 3.

--- upper and lower  
boundaries on ranges  
of  $\Psi\sqrt{s^2_{y.x}}$ .

PLATE XX





EXPLANATION OF PLATE XXI

Plots of  $\Psi$  AREA, dynamic stability, for three drops over entire range of disks.

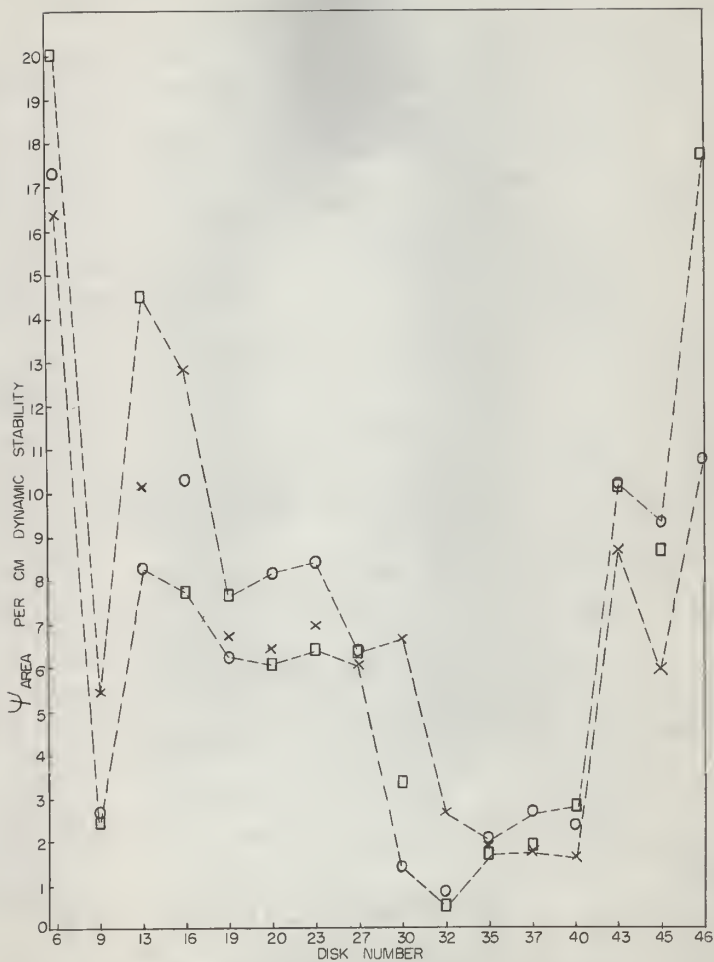
X Drop 1.

O Drop 2.

□ Drop 3.

--- upper and lower  
boundaries on ranges  
of  $\Psi$  AREA.

## PLATE XXI



EXPLANATION OF PLATE XXII

Plots of  $\psi s^2_{y.x}$ , dynamic stabilities for  
three drops over entire range of disks.

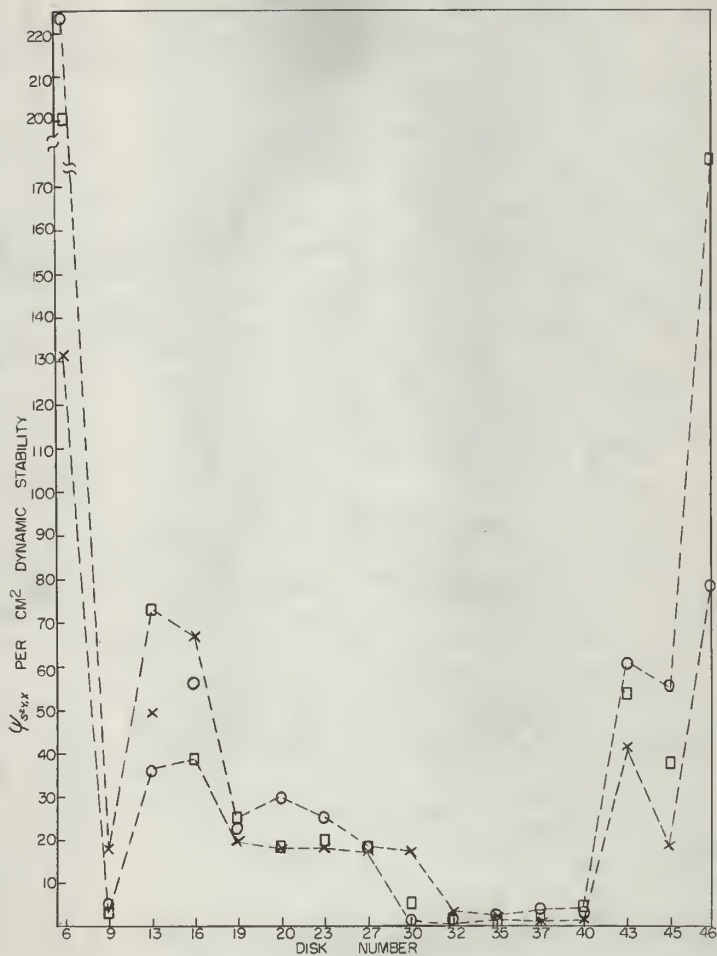
X Drop 1.

O Drop 2.

□ Drop 3.

( upper and lower  
boundaries on range  
of  $\psi s^2_{y.x}$ .

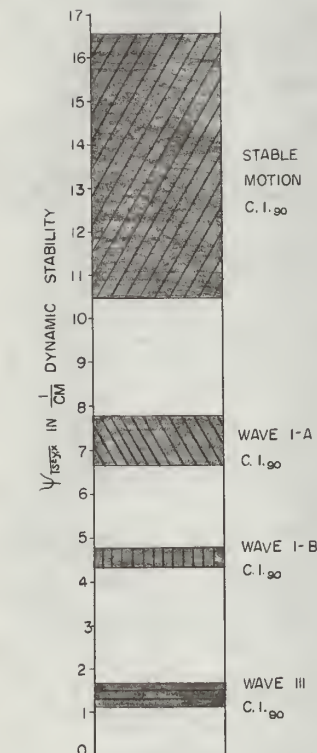
## PLATE XXII



#### EXPLANATION OF PLATE XXIII

Bar graph illustrating relationship of 90% confidence intervals of  $\psi \sqrt{s^2_{y.x}}$ , dynamic stabilities, of disks classified according to postulated types of motion.

## PLATE XXIII



#### EXPLANATION OF PLATE XXIV

Graph of properties of disks for three drops  
in relation to postulated types of motion  
assigned each disk.

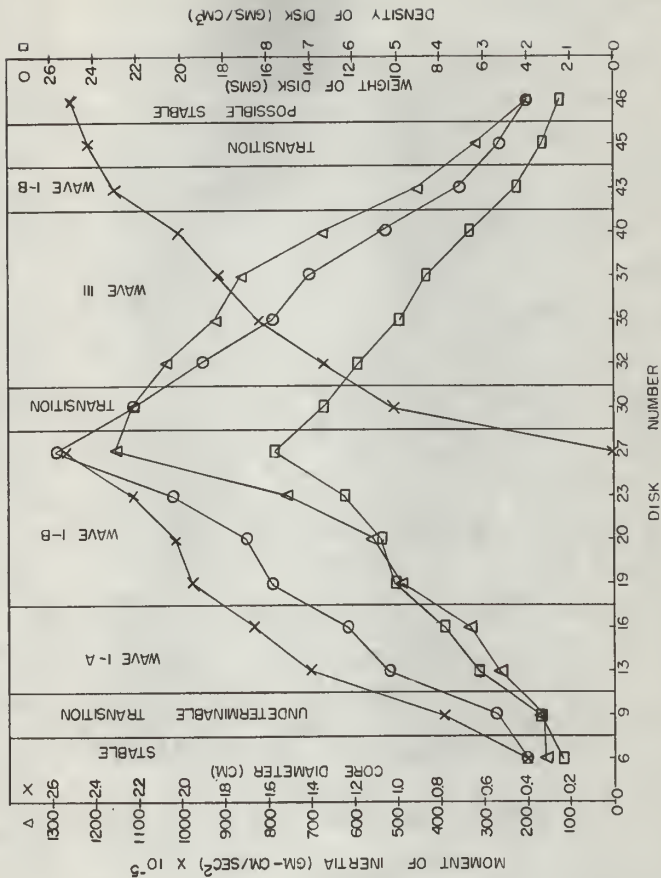
O weight of disk.

□ density of disk.

X core diameter of disk.

Δ moment of inertia of disk.

PLATE XXIV





## EXPLANATION OF PLATE XXV

Graph of Reynolds Numbers of disks for three drops in relation to postulated types of motion assigned each disk.

$$Re = \frac{D_s v \rho}{\mu}, \text{ where}$$

$D_s$  = diameter of sphere having same volume as disk in ft.

$v$  = velocity of disk in ft./sec.

$\rho$  = density of fluid in lb./ft.<sup>3</sup>

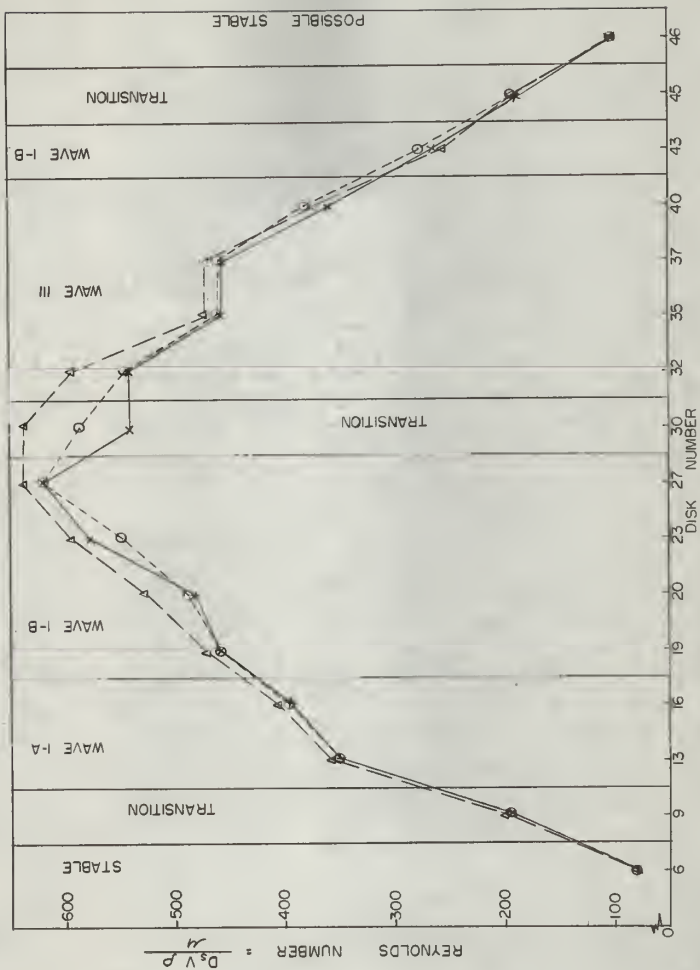
$\mu$  = viscosity of fluid in Bvu, lb./ft. sec.

X Drop 1.

□ Drop 2.

Δ Drop 3.

PLATE XXV



major part, by the position of the particle relative to the location of the reference wires and to the location of the camera. By moving the camera to a position further from the drop column the parallax was lessened by a calculable amount. However, the allowable distance between the camera and the tank was limited by the characteristics of the available telescopic lenses.

In summary extraneously induced disturbances of the descent of a disk and randomness of behavior in the descent of a disk provided incalculable and uncontrollable sources of disagreement between measured paths of descents. Measuring error was calculable but uncontrollable, while parallax was both calculable and controllable within equipment and camera limitations. Therefore, it remained to minimize parallax error to the degree where it was of the same or of a lesser magnitude than measuring error. Without altering the original locations of the reference wires this was accomplished by increasing the distance between the camera and the drop column to the maximum allowed by the available telescopic lenses. Confidence intervals on maximum measuring error when compared with maximum possible parallax error were found to be of the same range of magnitude. This comparison was presented in PARALLAX (3) in conjunction with a rigorous investigation of parallax.

Other sources of error were due to the non-uniformity of release of the disks and size limitations of the drop column. The former was minimized by disregarding an initial length of each descent while the latter was unavoidable for monetary reasons. Another source of error resulted from variations in the rate at which the camera filmed the descent. The speed of the camera was found to vary from 16.5 to 19.6

frames per second. This meant that the relative magnitude of each frame varied within a given descent. The assumption that all frames were equal to the same increment of time was not true. This variation was a characteristic of the spring and governor regulating the camera speed. It, also, depended upon how often and to what degree the photographer wound the camera. For this reason it was impossible to calculate the magnitude of this source of error. The solution to this problem lay in the statement, "An ounce of prevention is worth a pound of cure". However, "the ounce of prevention," consisted of a synchronizing motor attachment for the camera. Monetary limitations prevented this. However, after some consideration it was felt that this lack of uniformity in the camera speed was not nearly as critical as measuring and paralactic errors, since the variation of camera speed within any one descent was extremely small. This was borne out by the excellent agreement between the calculated dynamic stabilities of various descents of most disks.

The values of dynamic stability by each method -  $\Psi \sqrt{s^2_{y,x}}$ ,  $\Psi_{AREA}$ ,  $\Psi s^2_{y,x}$ , - were plotted in PLATES XX, XXI, and XXII, respectively. The method based upon  $\Psi \sqrt{s^2_{y,x}}$  was randomly chosen from these three for further consideration. In PLATE XIX,  $\Psi \sqrt{s^2_{y,x}}$  was plotted as the upper and lower limits of 90 per cent confidence intervals on the dynamic stabilities of each disk. This graph illustrated the statement that the true value of dynamic stability for each disk had a 90 per cent chance of lying between the individual limits described. This did not state that the true curve of dynamic stability had a 90 per cent chance of lying entirely between these limits formed by

connecting the individual limits. However, the limits so acquired by connecting these individual limits lead to the statement that each point of the true curve of dynamic stabilities had a 90 per cent chance of lying within these limits.

During the period of calculation of dynamic stabilities a completely isolated study was being conducted on the types of motion apparent in the disk descents. As was described earlier in this thesis under General Visual Observations on the Descent of Disks through Triethylene Glycol and Water each disk was assigned to one of three main types of motion. In PLATE XX the range of assigned motions were compared with the magnitudes of dynamic stabilities. In PLATE XXIV, a similar comparison was made between physical characteristics of the disks and assigned motions. In PLATE XXV, a like comparison was made between Reynolds numbers and assigned motions.

#### DISCUSSION OF RESULTS

With reference to Tables 7, 8, and 9 of the Appendix, it is seen that decided agreements between dynamic stabilities of the three descents of each disk were clearly observed for most of the disks. Disks not exhibiting this agreement were numbers 9, 30, 32, and 45. In these cases the disagreements, relative to the magnitudes of the dynamic stabilities, were large. Erratic behavior in the source of a measurable quantity was associated by this investigator with the inability to reproduce measurements of that quantity. Therefore, because of the visually observed erraticness of disks 30, 32, 35, 37, and 40 during their descents disagreement was expected, although only disks 30 and 32 actually exhibited

any marked diversion. This indicated some degree of uniformity of behavior within the erraticalness. Erraticalness in the descent of a disk was also associated with low dynamic stabilities. This was true; disks 32, 35, 27, and 40 had the lowest values of dynamic stability.

Dynamic stabilities,  $\Psi\sqrt{s^2_{y.x}}$ ,  $\Psi_{AREA}$ , and  $\Psi s^2_{y.x}$ , were plotted on PLATES XX, XXI, and XXII, respectively. From the plots of all three methods four levels of magnitudes of dynamic stability were apparent. Selecting  $\Psi\sqrt{s^2_{y.x}}$  for further study, it was found that disk 6, the most stable, formed the highest level. Disks 13, 16, and 43 formed the second level. Disks 19, 20, 23, and 27 formed the third. Disks 32, 25, 27, and 40 formed the lowest and final level. Disks 9, 30, and 45 had values of dynamic stability in more than one level. With only two values of dynamic stability it was improper to classify disk 46 in any of the levels.

The four levels were then compared with the postulated types of motion exhibited by the descending disks, Table 15, Appendix. This study, dealing with the types of motion, was conducted prior to and separate from the evaluation of dynamic stabilities. Therefore, the excellent agreement of the two was surprising. These postulated types of motion were compared with dynamic stabilities on the plot of  $\Psi\sqrt{s^2_{y.x}}$  against disk number, PLATE XX. The discrepancies caused by disks 9, 30, and 45 were resolved by considering these disks as transitional between two types of motion. Table 15, Appendix, substantiated this for disk 30, since this disk was found to exhibit more than one type of motion. Disk 43 was observed to exhibit wave I. Its values of dynamic stability did not, however, indicate to which subtype it belonged. Since one value

of dynamic stability on disk 43 fell into the wave I-A level and the other two into the wave I-B level, disk 43 was finally classified as transitional between wave I-A and wave I-B. Disk 9 presented the only outstanding discrepancy in that the observed motion, postulated as wave I, did not agree with the levels in which its dynamic stabilities fell - wave I-B and wave III. Therefore, disk 9 was classified as being in an undeterminable transition region.

Ninety per cent confidence intervals were calculated on the three values of dynamic stability for each disk, as presented in Table 3, Appendix. The close agreement between the confidence intervals on dynamic stability among various disks within a given type of fall indicated that the dynamic stabilities for these disks came from the same populations. The disks having comparable confidence intervals were found to form four separate groups. These four groups corresponded exactly to the four levels of dynamic stability earlier found to correspond to the postulated types of motion. Therefore, the values of dynamic stability for each disk, with the exceptions of 9, 30, and 45, fell into one of the postulated types of motion. Ninety per cent confidence intervals were then calculated on each group as presented in Table 4, Appendix. The limits on these groups, or levels, were then presented as a bar graph in PLATE XXIII. This illustrated the statement that the true value of dynamic stability for each of these groups had a 90 per cent chance of lying within the limits, indicating that each such level was distinct and separate from the others.

The next step was to make an attempt to correlate the properties of the disks and fluid to the assigned types of motion. The properties of



the disks were tabulated in Table 5, Appendix, and plotted in conjunction with the types of motion assigned each disk in PLATE XXIV. From Tables 7, 8, and 9 of the Appendix Re's were plotted in the same manner in PLATE XXV. From PLATE XXIV the only conclusive statement possible was that disks exhibited greater unetability as their momente of inertia became large relative to their weighte. Stable motion occurred at low weights, low moments of inertia, and low densities relative to the comparable properties for dieke which exhibited other postulated types of motion. Disk 6 exhibited stable motion. Finally, from PLATE XXV, the Reynolds numbers, based on  $D_s$ , for the three descents of disk 6 were less than 100. Disks exhibiting waves I-A, I-B, and III motion had Re's of 275 to 325, 225 to 650, and 300 to 625, respectively.

### CONCLUSIONS

The moet significant conclusion, based upon more than 41,000 meaasurements, was that dynamic etability of disks is measureable and therefore exists.

It was concluded that a disk, in a free-fall deecent, exhibited one of three postulated types of motion; i. e. stable, wave I, or wave III. It was further postulated that wave I motion consisted of subtypee; wave I-A and wave I-B. The distinction between these subtypes appeared in the measured dynamic stabilitiee; correeponding to the exact boundary predicted by the observations (See Table 15, Appendix). In water disks exhibited three main types of motion; i. e. wave, spiral, or disk rotation with diameter rotation, or combinations of these. For triethylene glycol



a definite relationship was found between these motions and dynamic stability,  $\psi \sqrt{s^2_{y,x}}$ .

Table 2: Relationship Between Types of Motion and  $\psi \sqrt{s^2_{y,x}}$ .

Disk number	Postulated type of motion	Range of dynamic stabilities, 1/cm.
6	Stable	11.49 to 14.92
13, 16, and 43	Wave I-A	6.02 to 8.55
19, 20, 23, and 27	Wave I-B	4.15 to 5.46
32, 35, 37 and 40	Wave III	0.44 to 2.01

Each range of dynamic stabilities corresponding to a particular type or subtype of motion was distinct and separate from any of the others. By applying confidence intervals to these ranges it was shown that the true value of dynamic stability for each type of motion had a 90 per cent chance of lying within limits which did not overlap the limits of other types of motion.

Actually, three methods,  $\psi \sqrt{s^2_{y,x}}$ ,  $\psi s^2_{y,x}$ , and  $\psi_{AREA}$  were used in calculating dynamic stability. The method based upon  $\psi \sqrt{s^2_{y,x}}$  was randomly chosen from these three for further consideration.

From a comparison of physical properties of the disks against the types of motion into which each disk fell it was found that a low dynamic stability was associated with a high moment of inertia relative to the weight of the disk. This is summarized in PLATE XXIV.

From a comparison of Reynolds numbers for each disk with the types of motion of each disk it was concluded that a  $Re$ , based on  $D_g$ , of one hundred or less was associated with stable motion. Disk 6, the only disk which clearly exhibited stable motion, was the lightest one and it also had the least density and moment of inertia of all those investigated.

These conclusions were all restricted to disks which descended in triethylene glycol at 88 to 90°F at Re, based on  $D_s$ , ranging from 78 to 640.

#### RECOMMENDATIONS

The following are recommended for immediate investigation:

1. Paralactic error may be noticeably reduced by moving the reference wires to a position in contact with the drop column. This suggests an investigation of the positioning of the top reference wire immediately above the release mechanism and the bottom reference wire inside the column near the bottom.

2. Parallax may be further reduced by removing the camera further from the drop column, thus suggesting that an investigation be made of telescopic lenses allowing this increased distance.

3. Recent films of disk descents revealed that a suspension of rust pigments in the triethylene glycol produced more distinct pictures of the disks. Other pigments may prove even more satisfactory.

4. Error, although not significant when compared to paralactic and measuring errors, was introduced into the final values of dynamic stability due to the non-uniformity of the camera speed. It is possible to rectify this by the use of a synchronizing motor. As paralactic and measuring errors are decreased this consideration will become important.

5. Measuring errors and measuring time could be noticeably decreased by moving the dark room and projector to a location supported by concrete flooring.

6. The size of the drop column should be appreciably increased. An increased diameter would lessen wall effects, while both an increased diameter and length would allow a more thorough study of disks exhibiting wave III motion.

The following are recommended for long-range investigation:

1. An analysis should be made of the possibility of using a radioactive source to expose film strips attached to the drop column to obtain a continuous picture of the disk descent.

2. A survey should be made of the possibility of employing either electric eyes or magnetic coils along the length of the column, such that with the addition of an appropriate recorder, readings of particle height may be directly obtained.

3. A study should be made of drop tanks other than cylindrical in shape.

Statements could be made concerning what next should be investigated, but dynamic stability has revealed such a fertile field as to make any recommendations for future work little more than twigs in an ocean, therefore, such concern will be left to the interests of future investigators.

#### ACKNOWLEDGMENTS

Some of the early experimental work which contributed to this thesis was conducted by Karl Mohn and Roger D. Allen, former students at Kansas State University, under the supervision of Assistant Professor Raymond C. Hall. The computer programs included in this thesis were

prepared by Dr. Thomas S. Parker and Thomas L. Hamilton of the IEM 650, Kansas State University. Both Dr. Parker and Mr. Hamilton, afforded this author much of their valuable time in supervising the evaluation of dynamic stability in the IEM 650. The disks used in this study were percision constructed by Mr. Andy Andereon of Building and Repair, Kaneas State University.

The most important contribution was certainly that of Profeseor Hall, whose patience, pedagogical judgement, and interest removed untold pitfalls to the successful completion of this work.

Another extremely important contribution was that of the author's wife, Francee B. Himes, who provided a never ending source of inepiration, patience and understanding.

A most deserving acknowledgment is, also, given to Dr. Henry T. Ward, Professor and Head of the Chemical Engineering Department.

The funde making this work possible were provided by the Agri-cultural Experiment Station of Kansae State University.

## REFERENCES

- (1) Becker, H. A.  
The effects of shape and Reynolds number on drag in the motion of a freely oriented body in an infinite fluid. The Canadian Journal of Chem. Engg., 85. April, 1959.
- (2) Dow Glycols, The Dow Chemical Company. Midland, Michigan. 1947.
- (3) Himes, B. L., Sr.  
Parallax and moments of inertia of disks employed in study. Unpublished report, Dept. of Chemical Engineering, Kansas State University, 1959.
- (4) Mickley, H. S., T. K. Sherwood, and C. E. Reed. Applied Mathematics in Chemical Engineering. New York: McGraw - Hill Book Company, Inc., 1957.
- (5) Miyagi, Otogoro  
Motion of an air bubble rising in water. Philosophical Magazine and Journal of Science. London. L: 125. 1925.
- (6) Pettyjohn, E. S. and E. B. Christiansen  
Effect of particle shape on free-settling rates of isometric particles. Chem. Engg. Progress. 44: 169. 1948.
- (7) Squires, Lombard and Walter Squires Jr.  
The sedimentation of thin disks. Transitions of the American Inst. of Chem. Engg. 33: 3. 1937.
- (8) Wadell, H.  
The coefficient of resistance as a function of Re. for solids of various shapes. Journal of the Franklin Institute. 217: 459. 1930.
- (9) Wetherall, E. W.  
The track of a flat solid through water. Nature. 60: 845. 1922.

## APPENDIX

Table 3. Calculation of G. I. 90 on Dynamic Stabilities

Disk Number	$\psi_{s^2}^2$	$y \cdot x$	1/cm	Drop 1	Drop 2	Drop 3	$\bar{x}_1$	Average	$\sum (x_{1k} - \bar{x}_1)^2$	$\sqrt{\sum S_m}$	90% Confidence Intervals
6	11.49	14.92	14.08	13.50	6.39	1.03	$10.49 \leq x_1 \leq 16.51$				
9	4.26	2.34	1.99	2.86	2.90	0.70	$0.82 \leq x_1 \leq 4.90$				
13	7.04	6.02	8.55	7.20	3.24	0.73	$5.07 \leq x_1 \leq 9.33$				
16	8.20	7.52	6.21	7.31	2.04	0.59	$5.60 \leq x_1 \leq 9.02$				
19	4.46	4.78	5.03	4.76	0.16	0.16	$4.29 \leq x_1 \leq 5.23$				
20	4.26	5.46	4.29	4.67	0.93	0.40	$3.52 \leq x_1 \leq 5.82$				
23	4.29	5.02	4.44	4.58	0.30	0.22	$3.93 \leq x_1 \leq 5.23$				
27	4.15	4.24	4.44	4.28	0.04	0.08	$4.03 \leq x_1 \leq 4.53$				
30	4.22	1.09	2.35	2.55	4.96	0.91	$0.00 \leq x_1 \leq 5.21$				
32	1.86	0.68	0.44	0.99	1.16	0.44	$0.00 \leq x_1 \leq 2.27$				
35	1.48	1.56	1.29	1.44	0.04	0.08	$1.21 \leq x_1 \leq 1.67$				
37	1.28	1.97	1.44	1.56	0.26	0.21	$0.95 \leq x_1 \leq 2.17$				
40	1.09	1.74	2.01	1.61	0.45	0.27	$0.81 \leq x_1 \leq 2.41$				
43	6.41	7.81	7.30	7.17	1.03	0.41	$5.96 \leq x_1 \leq 8.38$				
45	4.35	7.41	6.17	5.98	4.73	0.89	$3.39 \leq x_1 \leq 8.57$				
46*	---	8.85	13.16	11.00	9.28	2.15	$0.00 \leq x_1 \leq 24.57$				

$$D. F. = 3 - 1 = 2 \quad t_{0.10} = \pm 2.920$$

$$*D. F. = 2 - 1 = 1 \quad t_{0.10} = \pm 6.314$$

Table 4: Calculation of C. I. 90 on Dynamic Stabilities for each Type of Motion.

Motion	D. F.	Disk Number	$\psi \sqrt{s}$	$\frac{1}{cm.}$	Drop 1	Drop 2	Drop 3	$x_1$ Average	$\sum (x_{1k} - x_1)^2$	$\sqrt{s_m^2}$	90% Confidence Intervals (8)
Stable	2	6	11.49	14.92	14.08	13.50	6.39	1.03	$10.49 \leq x_1 \leq 16.51$		
Wave I-A	8	13 16 43	7.04 8.20 6.41	6.02 7.52 7.81	8.55 6.21 7.30	7.23	6.31	0.30	$6.68 \leq x_1 \leq 7.78$		
Wave I-B	11	19 20 23 27	4.46 4.26 4.29 4.15	4.78 5.46 5.02 4.24	5.03 4.29 4.44 4.44	4.57	1.83	0.12	$4.36 \leq x_1 \leq 4.78$		
Wave III	11	32 35 37 40	1.86 1.48 1.28 1.09	0.68 1.56 1.97 1.74	0.44 1.29 1.44 2.01	1.40	2.55	0.14	$1.15 \leq x_1 \leq 1.65$		
Transition -		9 30 45									
Insufficient Data -		46									
D.F. = Degrees of Freedom											

D.F. = Degrees of Freedom.



Table 5: Physical Characteristics of Disks.

Disk Number	Weight grams	Moment of Inertia (gm. mass - cm <sup>2</sup> )10 <sup>5</sup>	Thickness cm.	Disk Diameter cm.	Core Diameter cm.	Disk Density gm./cm. <sup>3</sup>
6	4.022	158	0.654	2.543	0.399	1.217
9	5.429	164	0.628	2.544	0.791	1.700
13	10.397	260	0.650	2.544	1.416	3.150
16	12.335	336	0.616	2.543	1.661	3.943
19	15.854	492	0.618	2.540	1.952	5.060
20	17.078	556	0.623	2.543	2.034	5.395
23	20.531	754	0.648	2.544	2.223	6.237
27	25.810	1154	0.650	2.540	2.540	7.843
30	22.113	1113	0.647	2.540	1.020	6.742
32	19.167	1034	0.636	2.541	1.341	5.950
35	15.835	921	0.627	2.540	1.646	4.986
37	14.186	864	0.645	2.540	1.822	4.341
40	10.703	677	0.635	2.538	2.067	3.333
43	7.210	452	0.639	2.539	2.306	2.230
45	5.425	319	0.639	2.542	2.421	1.674
46	4.016	206	0.634	2.542	2.503	1.250

Table 6: Average Times Required for Disks to Descend 100 cm. in Triethylene Glycol at Various Liquid Temperature.

Disk Number	Time in sec. : 66°F	Time in sec. : 86°F	Time in sec. : 88°F	Time in sec. : 89°F	Time in sec. : 90°F
	One Drop	Avg. Two Drops	Avg. Three Drops	Avg. Three Drops	Avg. Three Drops
6	12.9	11.1	10.9	11.2	10.5
7	---	6.5	6.5	6.4	6.1
8	---	6.1	6.1	6.1	6.1
9	4.2	4.3	4.5	4.6	4.5
10	---	3.7	3.6	3.7	3.7
11	---	2.9	3.0	3.0	2.9
13	2.2	2.3	2.5	2.5	2.5
16	2.2	2.1	2.2	2.2	2.2
19	1.9	1.9	1.9	1.9	1.9
20	1.8	1.8	1.8	1.8	1.7
23	1.5	1.5	1.5	1.6	1.5
27	1.3	1.3	1.4	1.4	1.4
30	---	1.7	1.6	1.5	1.4
32	1.5	1.6	1.6	1.6	1.5
35	1.6	1.8	1.9	1.9	1.9
37	2.0	2.0	1.9	1.9	1.9
40	2.1	2.3	2.4	2.3	2.4
43	3.0	3.2	3.3	3.2	3.5
45	4.2	4.6	4.6	4.7	4.6
46	10.3	9.1	8.9	8.9	8.7

Table 7: Dynamic Stabilities and Other Pertinent Final Results For Drop Two in Triethylene Glycol at 88°F.

Disk Number	$\theta$ : Frames : Read	$t_{\theta}$ : sec.	$\psi s^2$ : $1/\text{cm}^2$	$\psi s^2 y \cdot x$ : $1/\text{cm.}$	$\psi \sqrt{s^2}$ : $1/\text{cm.}$	$\psi$ Area : $1/\text{cm.}$	Re = Dvp : $\mu$	Re <sub>s</sub> Dvp = $\mu$
6	182	9.96	131.54		11.49	16.38	109	80
9	73	3.99	18.08		4.26	5.49	264	193
13	40	2.20	49.48		7.04	10.17	476	347
16	35	1.95	66.94		8.20	12.82	541	395
19	30	1.65	19.85		4.46	6.74	626	457
20	29	1.66	18.05		4.26	6.46	661	482
23	26	1.33	18.39		4.29	6.98	793	579
27	22	1.21	17.23		4.15	6.06	850	620
30	25	1.49	17.75		4.22	6.67	744	543
32	27	1.55	3.44		1.86	2.63	744	543
35	23	2.18	1.33		1.48	1.90	626	457
37	25	1.35	1.64		1.28	1.75	626	457
40	24	1.46	1.18		1.09	1.63	496	362
43	54	2.90	41.21		6.41	8.68	360	263
45	71	4.08	18.89		4.35	5.98	259	189
46								

$t_{\theta}$  = time required for particle to descend  $d_{\theta}$  ft.,  
or through  $\theta$  frames.

$D$  = diameter of disk.

$D_g$  = diameter of sphere with same volume as disk.

Table 8: Dynamic Stabilities and Other Pertinent Final Results for Drop Two in Triethylene Glycol at 89°F.

Disk Number	$\theta$ Frames Read	$t_\theta$ sec.	$\psi$ $s^2$ $1/cm^2$	$\psi \sqrt{s^2}$ $y \cdot x$ $1/cm$	$\psi$ Area $1/cm$	$Re =$ $\frac{D \psi p}{\mu}$	$Re_s =$ $\frac{D_s \psi p}{\mu}$
6	75	8.39	224.22	14.92	17.32	108	78
9	71	4.06	5.46	2.34	2.70	262	191
13	38	2.12	36.37	6.02	8.30	482	352
16	34	1.93	56.55	7.52	10.31	547	400
19	25	1.42	22.80	4.78	6.24	634	463
20	29	1.57	30.00	5.46	8.18	669	488
23	26	1.42	25.32	5.02	8.42	753	550
27	22	1.26	17.90	4.24	6.39	860	628
30	25	1.44	1.19	1.09	1.42	803	586
32	27	1.55	0.46	0.68	0.83	753	550
35	22	1.27	2.42	1.56	2.02	634	463
37	25	1.35	3.89	1.97	2.64	634	463
40	27	1.49	3.05	1.74	2.37	524	382
43	53	2.80	60.88	7.81	10.15	376	275
45	73	4.13	55.21	7.41	9.30	256	187
46	67	7.28	78.22	8.85	10.73	135	99

$t_\theta$  = time required for particle to descend  $d_\theta$   
ft., or through  $\theta$  frames.

$D$  = diameter of disk.

$D_s$  = diameter of sphere with same volume as disk.

Table 9: Dynamic Stabilities and Other Pertinent Final Results for Drop Three in Triethylene Glycol at 90°F.

Disk Number	$\theta$ : Frames : Read : :	$t_\theta$ : sec. : :	$\psi^2$ : $y \cdot x$ : $1/\text{cm}^2$ :	$\psi^2$ : $y \cdot x$ : $1/\text{cm}$ :	$\psi$ : Area : $1/\text{cm}$ :	$\frac{D_{vp}}{n}$ : :	$Re = \frac{D_{vp}}{n}$ : :	$Re_s = \frac{D_{sp}}{n}$ :
6	80	8.88	200.68	14.08	20.01	116	84	
9	71	3.92	3.95	1.99	2.42	279	204	
13	38	2.15	73.32	8.55	14.48	491	358	
16	30	1.70	38.84	6.21	7.75	558	407	
19	30	1.69	25.16	5.03	7.63	646	471	
20	28	1.49	18.47	4.29	6.08	722	527	
23	25	1.28	19.78	4.44	6.40	818	597	
27	19	1.21	19.82	4.44	6.36	876	640	
30	26	1.35	5.53	2.35	3.32	876	640	
32	25	1.44	0.19	0.44	0.51	818	597	
35	21	1.17	1.66	1.29	1.69	646	471	
37	33	1.83	2.09	1.44	1.86	646	471	
40	28	1.61	4.03	2.01	2.79	511	373	
43	59	3.43	53.14	7.30	10.13	350	256	
45	74	4.08	37.91	6.17	8.65	267	195	
46	71	7.67	175.41	13.16	17.68	141	103	

$t_\theta$  = time required for particle to descend  $d_\theta$  ft., or through  $\theta$  frames.

$D$  = diameter of disk.

$D_s$  = diameter of sphere with same volume as disk.

Table 10: Observations of diameter rotation on disks descending 100 cm. in triethylene glycol at  $90^\circ \pm 3^\circ$ .

Disk Number	1	2	3	4	5
6	/	/	/	/	/
7	/	/	/	—	—
8	/	/	/	—	—
9	/	/	/	/	/
10	/	/	/	—	—
11	/	/	/	—	—
13	/	/	/	/	/
16	/	/	/	/	/
19	/	/	/	/	/
20	—	$\bar{X}_{50}$	/	/	/
23	/	$\bar{X}_{100} - 150$	/	/	/
27	/	/	/	/	/
30	$\bar{X}_{200}$	/	/	/	/
32	/	—	/	/	/
35	/	/	/	/	/
37	$\bar{X}_{200}$	/	/	/	/
40	/	/	/	/	/
43	/	/	/	/	/
45	/	/	/	/	/
46	/	/	/	/	/

Top view of descending disk and corresponding designation in table.



Table 11: Observations of diameter rotation on disks descending 100 cm. in water at  $80^\circ \pm 3^\circ\text{F}$ .

Disk Number	1	2	3	4	5
1	X <sub>150</sub>	X <sub>450</sub>	X <sub>750</sub>	X <sub>700</sub>	X <sub>150</sub>
6	X <sub>450</sub>	X <sub>600</sub>	X <sub>1000</sub>	X <sub>700</sub>	X <sub>150</sub>
9	X <sub>900</sub>	X <sub>1800</sub>	X <sub>500</sub> to - 200°	X <sub>2000</sub>	X <sub>500</sub>
13	/	/	X <sub>900</sub> near bottom	X <sub>800</sub> near bottom	/
16	X <sub>200</sub>	X <sub>400</sub>	X <sub>500</sub> near bottom	X <sub>600</sub>	X <sub>100</sub>
19	X <sub>600</sub>	X <sub>1000</sub> to - 60°	/	X <sub>200</sub> to - 10°	X <sub>200</sub>
20	X <sub>500</sub> to - 20°	X <sub>1000</sub> to - 20°	/	/	/
23	X <sub>150</sub>	/	X <sub>450</sub> near bottom	X <sub>200</sub> to - 10° near bottom	X <sub>200</sub>
27	X <sub>500</sub>	/	/	/	/
30	/	/	/	/	/
32	/	/	/	/	/
35	/	/	X <sub>1000</sub>	X <sub>1000</sub>	X <sub>150</sub>
37	/	X <sub>500</sub> near bottom	/	/	/
40	/	X <sub>1000</sub> to - 10°	/	X <sub>500</sub> near bottom	X <sub>200</sub>
43	X <sub>500</sub> to - 25°	X <sub>400</sub>	X <sub>1000</sub> to - 20°	X <sub>500</sub> near bottom	X <sub>100</sub> to - 10°
45	X <sub>1700</sub>	X <sub>1000</sub> to - 180°	X <sub>2000</sub> near bottom	X <sub>1000</sub>	X <sub>100</sub> to - 30°
46	X <sub>1200</sub>	X <sub>1450</sub>	X <sub>2000</sub> near bottom	X <sub>1700</sub>	/

Top view of descending disk and corresponding designation table.



No descent  
=

Table 12: Measured Angle or Angles Between Vertical Plane Through Path of Disk Descent and a Reference Vertical Plane in a 100 cm. Deep Triethylene Glycol Column at  $90 \pm 30^\circ \text{F}$ .

Disk Number	:	Drop 1	:	Drop 2	:	Drop 3	:	Drop 4
6								
9		-60°		-40°		15°		60°
13		85°		-60°		-55°		-60°
16		55°		-50°		-35°		50°
19		60°		-45°		-53°		55°
20		-40°		-45°		47°		50°
23		50°		65°		-70°		-60°
27		35°		75°		-70°		0°
30		80°		-70°		50°		90°
32		-50° to -80°		-85°		70°		-85°
35		50°		-65° to -80°		80°		50°
37		40°		65°		60°		90°
40		-80°		-85°		75°		-60°
43		-60°		-45°		-90°		-55°
45				70°		-85°		-80°
46								



Table 13: Observed Motions of Disk Descending Approximately 100 cm. in Triethylene Glycol at  $90 \pm 3^\circ\text{F}$ .

Disk Number	Motion - Side View		
	Drop 1	Drop 2	Drop 3
6	Stable	Stable	Stable
9	Flutter	Wave	Flutter
13	Flutter	Wave	Wave
16	Small Wave	Wave	Wave
19	Wave	Wave	Wave with Flutter
20	Flutter	Wave	Wave with Flutter
23	Wave	Wave	Wave
27	Wave	Wave	Wave
30	Wave - Hit Wall	Wave - Hit Wall	Wave with Flutter
32	Wave - Hit Wall	Wave - Hit Wall	Wave - Hit Wall
35	Wave - Hit Wall	Wave - Hit Wall	Wave - Hit Wall
37	Wave - Hit Wall	Wave - Hit Wall	Wave - Hit Wall
40	Wave - Hit Wall	Wave - Hit Wall	Wave - Hit Wall
43	Small Wave	Flutter	Wave
45	Small Wave	Small Wave	Flutter
46	Stable	Stable	Stable

Table 13 (Concl.):

Disk Number	Motion - Top View	
	Drop 4	Drop 5
6	Stable	Stable
9	Oscillation	Large Flutter
13	Wave With Oscillation	Wave With Oscillation
16	w/o Diameter Rotation	w/o Diameter Rotation
19	Wave	Wave
20	Wave	Wave
23	Wave	Wave
27	Wave	Wave
30	Wave	Wave
32	Wave - Hit Wall	Wave - Hit Wall
35	Wave - Hit Wall	Wave - Hit Wall
37	Wave - Hit Wall	Wave - Hit Wall
40	Wave - Hit Wall	Wave - Hit Wall
43	Small Wave	Large Flutter
45	Very Small Wave	Flutter and Oscillation
46	Stable	w/o Diameter Rotation
		Stable

Table 14: Observed motions of disk descending approximately 100 cm. in water at  $80 \pm 3^\circ\text{F}$ .

Disk Number	Combined Top and Side Views of Particle Motion		
	Drop 1	Drop 2	Drop 3
1	Spiral	Spiral	Spiral
6	Spiral	Oscillation with Diameter	Spiral
9	Planar Wave to Spiral	Rotation in Planar Wave	Spiral
13	Planar Wave to Spiral	Planar Wave with Diameter	Planar Wave to Spiral
16	Flutter or Small	Rotation to Spiral	Non-Planar
19	Planar Wave	Planar Flutter	Wave
20	Planar Wave to Spiral	Small Planar Wave	Small Planar Wave
23	Completely Erratic	with Rotation Near Bottom	Small Planar Wave
27	Completely Erratic	Planar Flutter	Small Planar Wave
30	Hit Side	Planar Flutter	Planar Wave - Hit Side
32	Planar Wave	Planar Wave - Hit Side	Planar Wave - Hit Side
35	Hit Side	Hit Side	Hit Side
37	Hit Side	Planar Wave	Rolling Motion
40	Planar Wave to Spiral	Hit Side	Hit Side
43	to Planar Wave	Planar Wave	Rolling Motion
45	Planar Wave to Spiral	Hit Side	Hit Side
46	Planar Wave to Spiral	Completely Erratic	Rolling Motion
		Planar Wave	Planar Wave
		Planar Wave	Planar Wave
		Planar Wave to Spiral	Planar Wave
		Planar Wave to Spiral	Planar Wave to Spiral
		Planar Wave to Spiral	Planar Wave to Spiral

Table 14 (Concl.):

		Top View of Particle Motion	
Disk	:	Drop 2	:
Number	:		
1	:	Started Planar Oscillation To Spiral	Spiral
6	:	Oscillation with Diameter Rotation in Planar Wave	Planar Flutter to Non- Planar to Spiral
9	:	Non - Planar Wave	Planar Flutter to Spiral
13	:	Planar Flutter	Planar Flutter to Spiral
16	:	Flutter or Small Planar Wave	Planar Flutter
19	:	Planar Flutter	Planar Wave
20	:	Planar Flutter	Planar Wave
23	:	Planar Wave with	Planar Wave
27	:	Rotation of Plane Near Bottom	Planar Wave
30	:	Small Planar Wave	Planar Wave
32	:	Hit Side	Hit Side
35	:	Rolling Motion	Rolling Motion
37	:	Small Planar Wave	Hit Side
40	:	Small Planar Wave	Non - Planar Wave
43	:	Small Planar Wave	Hit Side
45	:	Small Planar Wave	Non - Planar Wave
46	:	Planar Wave	Planar Wave
	:	Planar Wave to Spiral	Planar Wave
	:	Planar Wave to Spiral	Planar Flutter to Spiral
	:	Planar Wave to Spiral	Planar Flutter to Spiral

Table 15: Observations Obtained from Sketches of Movies and Movies of Disk Descents in Triethylene Glycol at  $90 \pm 3^\circ\text{F}$ .

Disk Number	Motion		
	Drop 1	Drop 2	Drop 3
6	Stable	Stable	Stable
9	Wave I	Wave I	Wave I
13	Wave I	Wave I	Wave I
16	Wave I	Wave I	Wave I
19	Wave I (I-B)	Wave I (I-B)	Wave I
20	Wave I (I-B)	Wave I (I-B)	Wave I
23	Wave I (I-B)	Wave I (I-B)	Wave I
27	Wave I (I-B)	Wave I (I-B)	Wave I
30	Wave I (I-B)	Wave III	Wave I
32	Wave III	Wave III	Wave III
35	Wave III	Wave III	Wave III
37	Wave III	Wave III	Wave III
40	Wave III	Wave III	Wave III
43	Wave I	Wave I	Wave I
45	Wave I	Wave I	Wave I
46	Stable	Stable	Stable

### Triethylene Glycol as the Fluid Media

Triethylene Glycol was a relatively new compound commercially at the initiation of this research. Its high boiling point, low volatility, low toxicity, low corrosion rate, and high stability made it ideal as a high boiling solvent. These same properties plus its high viscosity at room temperatures and its transparency made it similarly ideal for the study of particle motions. These properties are presented in tables and graphs published in Dow Glycols (2).

Because of the hygroscopic property of glycols, it was imperative to determine whether changes in the water content of the fluid media had occurred between the times of utilization of the fluid media. As an example, an evaluation of the viscosity of the fluid media required the determination of a density value by actual measurement. This, in turn, provided a step toward the determination of the percentage of pure triethylene glycol in the fluid media. The density of the fluid media was obtained by the utilization of a volumetric flask, scales, and a properly selected sample. From this density and the density of pure water at the same temperature, 25°C, the specific gravity of the fluid media at 25°C was calculated. With this information the percentage of Triethylene Glycol was found from a plot of specific gravity, 25°C/25°C, versus per cent glycol by weight (2). In turn the viscosity at any desired temperature was found from a plot of absolute viscosities of aqueous Triethylene Glycol solutions versus Temperature (2). It was found that over a twelve month period with the proper protection of the fluid from the atmosphere no appreciable change occurred in the water content of the approximately 200 lbs. of fluid. It remained approximately 100% pure Triethylene Glycol.

## TABLE OF NOMENCLATURE

1.  $s_{y.x}^2$  = mean square deviation from regression.
2.  $s_{y.x}$  = sample standard deviation from regression =  $\sqrt{s_{y.x}^2}$ .
3.  $s_{y.x\theta}^2 = s_{y.x}^2 (F_\theta)$ .
4.  $\theta$  = number of motion picture frames through which disk is measured.
5.  $\Psi_{s_{y.x}^2}$  = dynamic stability based on  $s_{y.x}^2 = \frac{1}{s_{y.x}^2}$ .
6.  $\Psi_{\sqrt{s_{y.x}^2}}$  = dynamic stability =  $\sqrt{\Psi_{s_{y.x}^2}}$ .
7.  $\Psi_{\text{AREA}}$  = dynamic stability based on area between least squares curves and path of disk descent, per unit time.
8.  $\Psi'$  = dynamic unstability,  $\Psi_{s_{y.x}^2} \frac{1}{s_{y.x}^2}$ ,  $\Psi_{\sqrt{s_{y.x}^2}} \frac{1}{\sqrt{s_{y.x}^2}}$ , or  $\Psi_{\text{AREA}} \frac{1}{\Psi_{\text{AREA}}}$ .
9.  $d_{y.x}$  = absolute deviations from regression =  $s_{y.x} (\theta-2)$ .
10.  $t_{100}$  = time required for disk to descend 100 cm.
11.  $t_\theta$  = time required for disk to descend  $\theta$  frames.
12.  $L_H$  = measured height of top reference line, relative to base line.
13.  $L_P$  = measured height of disk, relative to base line.
14.  $L_L$  = measured height of bottom reference line, relative to base line.
15.  $T.L. = \frac{100}{L_H - L_L}$  = factor for adjusting relative values to true values.
16.  $h_i = (L_P - L_L) \left( \frac{100}{L_H - L_L} \right)$  = true height of particle above reference line.
17.  $n$  = number of sample under statistical consideration.
18.  $k$  =  $y$  intercept of least squares line.
19.  $j$  = slope of least squares line.
20.  $I$  = moment of inertia of disk.

## TABLE OF NOMENCLATURE (concl.)

- 21.  $Re$  = Reynolds number based on  $D = \frac{DV\rho}{\mu}$  .
  - 22.  $D$  = diameter of disk in feet.
  - 23.  $V$  = average velocity of disk in feet per second.
  - 24.  $\rho$  = density of disk in pounds/feet cube.
  - 25.  $\mu$  = viscosity in British viscosity units.
  - 26.  $Re_s$  = Reynolds number based on  $D_s = \frac{D_s V_s \rho}{\mu}$  .
  - 27.  $D_s$  = diameter of sphere having same volume as disk.
  - 28.  $t_{0.05}$  = student's  $t$  for 95 per cent confidence interval = 2.571.
  - 29.  $F$  = conversion factor of frames to seconds.
  - 30.  $x$  = time axis
  - 31.  $y$  = height axis
- } on plot of disk path (PLATE XIV).



## TABLE OF DEFINITIONS

1. Drop referred to a sum of single descents for all disks investigated.
2. Descent referred to a movement downward of a single disk.
3. Dynamic stability referred to the stability of a disk during free fall in a container of infinite proportions.
4. Parallax was defined as a distortion of the measured values of the differences between consecutive positions of a descending disk.
5. Consecutive positions referred to consecutive frames on the film of the descent of a disk where these positions, or frames, were separated by a constant time increment.
6. Confidence intervals were defined as the limits between which a true value had a certain probability of existing.
7. Dark room was the room containing the equipment for measuring disk heights on the films of the descent.
8. Least squares line was the best possible straight line through a plot of the path of the particle descent which minimizes the squares of the deviations.
9. Measuring error referred to the error resulting from the measurements of disk height made on the films of the disk descent.
10. Extraneous disturbances referred to disturbances caused by movements outside of the measuring area.
11. Stable Motion (See PLATE XVII).
12. Wave I (See PLATE XVII).
13. Wave II (See PLATE XVIII).
14. Wave III (See PLATE XVIII).

## TABLE OF DEFINITIONS (concl.)

15. Oscillation (See PLATE XVIV).
16. Spiral (See PLATE XVIV).
17. Agile plastic stars were star shaped pieces of plastic approximately  $1\frac{1}{2}$  inches in diameter and  $\frac{1}{2}$  inch thick used normally to minimize evaporation from a liquid surface.
18. Moment of Inertia referred to integral from center of the core to outside of the core of the differential mass times the arm squared plus integral from outside of core to outside of rim of the differential mass times arm squared, where the arm is the distance from the center of the disk to each differential mass.

IBM 650 PROGRAMS

for

COMPUTATION OF DYNAMIC STABILITY

SOAP  
Output  
Of A

FORTTRANSIT  
Program  
For

$\psi^2_{y.x}$  And Least Squares Line

Programmed

by

Mr. Thomas L. Hamilton

IBM 650

Kansas State University

Program is preceded by a SOAP package  
deck from FORTTRANSIT II.

E00BT REG Y0002 0054  
 NZU EXITT EXITT  
 STU SIGNS  
 RAM B003  
 RAU H002  
 STU C0002  
 RSL B002  
 ALO PHPHI  
 SRT 0004  
 ALO 0000  
 SKIPS STL TEMPO  
 TRIES RAU COUNT  
 SLT 0002 TEMPO  
 STU COUNT  
 RAU SIGNS  
 BMI POSIT  
 RBU COUNT  
 RAU COUNT  
 SS 0000  
 E00BU BTO EXITT  
 NZU EXITT  
 STU SIGNB  
 RAM B003  
 RAU H002  
 SCT 0000  
 SUP FIFIV  
 STU B002  
 RAU ASSOL  
 RAU SIGNS  
 BMI PLUSS  
 RSU ARCOL  
 RAU ARCOL  
 OO 0000  
 1 ARE ROOT  
 STU BEXT  
 BMI SERR  
 NZE SEXT  
 STU SA  
 FAO B10  
 FMP SHAF  
 STU SBAV  
 RAU SA  
 FOV SBAV  
 FAO SBAV  
 FMP SHAF  
 FSB SBAV  
 NZU SR  
 BMI SR  
 FAO SBAV  
 STU SBAV  
 RAU SBAV  
 8R SBAV  
 8ERR HLT 0000  
 SHAF 50 0000  
 SIO 10 0000  
 ES001 OO 0000  
 LAAAA RAL EZ001  
 8TL W0002  
 RAL EZ002  
 8TL W0003  
 RAL EZ003  
 8TL W0004  
 RAL EZ004  
 8TL W0005  
 RAL EZ005  
 8TL W0006  
 RAL EZ006  
 LOO LABAA  
 OO 0000  
 E8002 RAU EZ007  
 LABAA STU Y0007  
 E8003 OO 0000  
 LABAA RAU EZ007  
 8TL Y0008  
 E8004 OO 0000  
 LABAA RAU EZ007  
 STU Y0009  
 E8005 OO 0000  
 LAEAA RAU EZ007  
 STU Y0010  
 E8006 OO 0000  
 LAFAA RAU EZ007  
 STU Y0011  
 E8007 OO 0000  
 LABAA RAA Y0009  
 LOO R0005  
 STB Y0032  
 ET033 OO 0000  
 ES008 OO 0000  
 LABAA RAB Y0000

0000 00 0000 0000  
 0104 24 0057 0060  
 0060 44 0063 0057  
 0063 21 0068 0071  
 0071 67 8003 0079  
 0079 60 8002 0087  
 0087 30 0002 0093  
 0093 21 0088 0001  
 0001 66 8002 0059  
 0059 15 0062 0067  
 0067 30 0004 0077  
 0077 15 0080 0085  
 0080 30 0000 0103  
 0085 20 0089 0092  
 0092 60 0098 0153  
 0153 35 0002 0099  
 0103 21 0098 0101  
 0101 60 0068 0073  
 0073 46 0076 0127  
 0076 61 0098 0057  
 0127 60 0098 0057  
 0062 55 0000 0000  
 0154 24 0057 0110  
 0110 44 0113 0057  
 0113 21 0118 0121  
 0121 67 8003 0129  
 0129 60 8002 0137  
 0137 36 0000 0109  
 0109 10 0112 0117  
 0117 11 8002 0075  
 0075 21 0130 0083  
 0083 60 0118 0123  
 0123 46 0126 0177  
 0126 61 0130 0057  
 0177 60 0130 0057  
 0112 00 0000 0060  
 0000 00 0000 0060  
 0204 24 0107 0160  
 0160 46 0163 0064  
 0064 45 0168 0107  
 0168 21 0072 0125  
 0125 32 0078 0055  
 0055 39 0058 0108  
 0108 21 0162 0065  
 0065 60 0072 0227  
 0227 34 0162 0212  
 0212 32 0162 0139  
 0139 39 0058 0158  
 0158 33 0162 0189  
 0189 44 0143 0094  
 0143 46 0096 0094  
 0096 32 0162 0239  
 0239 21 0162 0065  
 0094 60 0162 0107  
 0163 01 0000 0107  
 0058 50 0000 0050  
 0078 10 0000 0051  
 0000 00 0000 0001  
 1999 65 0102 0157  
 0157 20 1968 0171  
 0171 65 0074 0179  
 0179 20 1969 0122  
 0122 65 0175 0229  
 0229 20 1970 0173  
 0173 65 0176 0081  
 0081 20 1971 0124  
 0124 65 0277 0131  
 0131 20 1972 0225  
 0225 65 0128 0133  
 0133 69 0086 1913  
 0000 00 0000 0002  
 0086 60 0289 0193  
 0193 21 0000 0181  
 0000 00 0000 0003  
 0181 60 0289 0243  
 0243 21 0089 0082  
 0000 00 0000 0004  
 0082 60 0289 0293  
 0293 21 0030 0183  
 0000 00 0000 0005  
 0183 60 0289 0343  
 0343 21 0031 0084  
 0000 00 0000 0006  
 0084 60 0289 0393  
 0393 21 0038 0135  
 0000 00 0000 0007  
 0135 80 0001 0091  
 0091 69 8005 0097  
 0097 24 0033 0136  
 0000 00 0100 0303  
 0000 00 0000 0008  
 0136 82 0001 0142

	LOO	8006			0142	69	8006	0148
	STO	Y0033	LAIAA		0148	24	0034	0187
ET038	OO	0000	LAIAA		0000	00	0100	0308
ES009	OO	0000	LAIAA		0000	00	0000	0009
LAIAA	RAL	EZ008			0187	65	0090	0095
	STL	W0002			0095	20	1968	0221
	RAL	EZ009			0221	65	0174	0279
ES010	LDD	LAJAA	EO0AQ		0279	69	0132	1913
LAJAA	OO	0000	LAJAA		0000	00	0000	0100
	RAC	Y0000			0132	81	0001	0088
	LOO	8007			0088	69	8007	0144
	STO	Y0034	LAKAA		0144	24	0035	0138
ET048	OO	0000	LAKAA		0000	00	0100	0408
ES011	OO	0000	LAKAA		0000	00	0000	0101
LAKAA	RAU	6019			0138	60	6019	0329
	STU	Y0035	LALAA		0329	21	0036	0339
ES012	OO	0000	LALAA		0000	00	0000	0102
LALAA	RAU	Y0035			0339	60	0036	0141
	STU	ACC			0141	21	0000	0203
	RAL	8006			0203	65	8006	0061
	STL	W0001			0061	20	1967	0070
	RAU	8007			0070	60	8007	0327
	MPY	EZ011			0327	19	0120	0100
	ALO	W0001			0100	15	1967	0271
	STL	W0001			0271	20	1967	0120
	RSL	EZ011			0120	66	0180	0185
	ALO	W0001			0185	15	1967	0321
	SLT	Y0003			0321	35	0004	0231
	ALO	LALAN			0231	15	0134	0389
	LOO	ACC	8002		0389	69	0000	8002
LALAN	STO	Y0000	LAMAA		0134	24	0001	0254
ES000	OO	0000	LAMAA		0000	00	0000	0000
LAMAA	AXC	Y0000			0254	58	0001	0210
	RSL	8007			0210	66	8007	0167
	STD	Y0034			0167	24	0035	0188
	ALO	EZ010			0188	15	0191	0145
	BMI	LANAA	ET048		0145	46	0198	0138
EB000	OO	0000	LANAA		0000	00	0000	0000
LANAA	AXB	Y0000			0198	52	0001	0304
	RSL	8006			0304	66	8006	0111
	STD	Y0033			0111	24	0034	0237
	ALO	Y0025			0237	15	0026	0281
	BMI	LA0AA	ET038		0281	46	0184	0187
EB013	OO	0000	LA0AA		0000	00	0000	0103
LA0AA	RAU	EZ007			0184	60	0289	0443
	STU	Y0036	LAPAA		0443	21	0037	0140
EB014	OO	0000	LAPAA		0000	00	0000	0104
LAPAA	RAU	EZ007			0140	60	0289	0493
	STU	Y0037	LA0AA		0493	24	0038	0241
EB015	OO	0000	LA0AA		0000	00	0000	0105
LA0AA	RAB	Y0000			0241	82	0001	0147
	LOO	8006			0147	69	8006	0253
	STO	Y0033	LARAA		0253	24	0034	0287
ET091	OO	0000	LARAA		0000	00	0100	0901
ES016	OO	0000	LARAA		0000	00	0000	0106
LARAA	RSU	4013			0287	61	4013	0217
	FAO	4001			0217	32	4001	0377
	STU	W0001			0377	21	1967	0170
	RSU	4013			0170	61	4013	0267
	FAO	4007			0267	32	4007	0233
	FMP	EZ012			0233	39	0186	0236
	FOV	W0001			0236	34	1967	0317
	LOO		EO0AA		0317	69	0220	1700
	STU	Y0038	LASAA		0220	21	0039	0192
EB017	OO	0000	LASAA		0000	00	0000	0107
LASAA	RAU	Y0038			0192	60	0039	0543
	FAO	Y0036			0543	32	0037	0203
	LOO		EO0AA		0203	69	0066	1700
	STU	Y0036	LATAA		0066	21	0037	0190
EB018	OO	0000	LATAA		0000	00	0000	0108
LATAA	RAU	Y0038			0190	60	0039	0593
	FMP	Y0038			0593	39	0039	0439
	FAD	Y0037			0439	32	0038	0115
	LDD		EO0AA		0115	69	0218	1700
	STU	Y0037	LAAUA		0218	21	0038	0291
EB000	OO	0000	LAAUA		0000	00	0000	0000
LAAUA	AXB	Y0000			0291	52	0001	0197
	RSL	8006			0197	66	8006	0105
	STD	Y0033			0105	24	0034	0337
	ALO	Y0025			0337	15	0026	0331
	BMI	LA0AA	ET091		0331	46	0000	0287
ES019	OO	0000	LAVAA		0000	00	0000	0109
LAVAA	RSU	EZ013	LAVAO		0234	61	0037	0341
LAVAH	RAL	Y0025			0354	65	0028	0381
	STU	ACC	EO0AE		0381	69	0044	0326
LAVAU	STU	ACC	LAVAB		0341	21	0000	0354
LAVAE	FAO	ACC			0284	32	0000	0427
	STU	W0001			0427	21	1967	0270
	RAL	Y0025			0270	65	0028	0431
	STL	W0004			0431	20	1970	0273
	RAL	EZ014	LAVAK		0273	65	0276	0481

LAVAJ	RAU	Y0036	LAVAJ	0404	60	0037	0391
LAVAK	STL	ACC	LAVAJ	0481	20	0000	0404
LAVAL	LDD		EOOAL	0391	69	0194	1850
	STU	ACC		0194	21	0000	0303
	RAL	W0004		0303	65	0170	0320
	LDD		EOOAE	0275	69	0041	0295
	STU	W0004		0178	21	1970	0323
	RAU	ACC		0323	60	0000	0155
	FOV	W0004		0155	34	1570	0220
	RSU	B0003		0320	61	0003	0477
	FAD	Y0037		0477	32	0038	0165
	FOV	W0001		0165	34	1967	0367
	LDD		EOOAA	0367	69	0370	1700
	STU	Y0039	LAWAA	0370	21	0040	0643
ES020	OO	0000	LAWAA	0000	00	0000	0200
LAWAA	RAL	Y0025	LAWAC	0643	65	0026	0531
LAWAB	RAU	Y0039	LAWAU	0454	60	0040	0195
LAWAC	LDD	LAWAB	EOOAF	0531	69	0454	1752
LAWAU	FOV	ACC		0195	34	0000	0150
	LDD		EOOAA	0150	69	0353	0204
	STU	Y0040	EOOAA	0353	69	0056	1700
ES021	OO	0000	LAXAA	0056	21	0041	0244
LAXAA	RAL	Y0025	LAXAA	0000	00	0000	0201
LAXAR	RAU	Y0036	LAXAC	0244	65	0026	0581
LAXAC	LDD	LAXAB	EOOAF	0504	60	0037	0441
LAXAU	FOV	ACC		0581	69	0504	1752
	LDD		EOOAA	0441	21	0000	0200
	STU	Y0041	LAYAA	0200	69	0403	1700
ES022	OO	0000	LAYAA	0403	21	0042	0245
LAYAA	RAU	Y0040		0000	00	0000	0202
	FMP	Y0026		0245	60	0041	0345
	FAD	Y0041		0295	39	0027	0527
	LDD		EOOAA	0527	32	0042	0069
	STU	Y0042	LAZAA	0069	69	0172	1700
ES023	OO	0000	LAZAA	0172	21	0043	0146
LAZAA	RAU	Y0040		0000	00	0000	0203
	FMP	Y0026		0146	60	0041	0345
	RSU	B0003		0345	39	0027	0577
	FAO	Y0041		0577	32	0042	0119
	LDD		EOOAA	0235	69	0222	1700
	STU	Y0043	LBAAA	0119	21	0044	0247
ES024	OO	0000	LBAAA	0222	00	0000	0204
LBAAA	RAL	B0005	EOOAF	0000	65	0005	0205
	LDD		LBHAA	0205	69	0208	1752
	STU	Y0044	LBHAA	0208	21	0045	0248
ES025	OO	0000		0000	00	0000	0205
LBAAA	RAU	Y0044		0248	60	0045	0209
	FAD	Y0027	EOOAA	32	0028	0255	
	LDD		LBAAA	0255	69	0258	1700
	STU	Y0027	LBAAA	0258	21	0028	0631
ES026	OO	0000	LBAAA	0000	00	0000	0206
LBAAA	RAU	Y0044		0631	60	0045	0149
	FMP	Y0044		0149	39	0045	0395
	FAO	Y0028		0395	32	0029	0305
	LDD		EOOAA	0305	69	0308	1700
	STU	Y0028	LBAAA	0308	21	0029	0182
ES027	OO	0000	LBAAA	0000	00	0000	0207
LBAAA	RAU	Y0041		0182	60	0042	0297
	FAD	Y0029		0297	32	0030	0207
	LDD		EOOAA	0207	69	0260	1700
	STU	Y0029	LBAAA	0260	21	0030	0283
ES028	OO	0000		0000	00	0000	0208
LBAAA	RAU	Y0041		0283	60	0042	0347
	FMP	Y0041		0347	39	0042	0242
	FAO	Y0030		0242	32	0031	0257
	LDD		EOOAA	0257	69	0310	1700
	STU	Y0030	ES029	0310	21	0031	0199
ES029	OO	0000	LBAAA	0000	00	0000	0209
LBAAA	RAU	Y0041		0199	60	0042	0301
	FMP	Y0044		0447	39	0045	0445
	FAO	Y0031		0445	32	0032	0159
	LDD		EOOAA	0159	69	0262	1700
	STU	Y0031	LBHAA	0262	21	0032	0134
ES030	OO	0000	LBHAA	0000	00	0000	0300
LBHAA	RAU	Y0042		0334	60	0043	0497
	LDD		EOOBT	0497	69	0503	0104
	STU	Y0045	LBIAA	0503	21	0046	0249
ES031	OO	0000	LBIAA	0000	00	0000	0301
LBIAA	RAU	Y0043	EOOBT	0249	60	0044	0299
	LDD			0299	69	0355	0104
	STU	Y0046		0355	21	0047	0350
	RAU	Y0041		0350	60	0042	1533
	LDD		EOOBT	0503	69	1821	0104
	STU	Y0041	LBIAA	1821	21	0042	1529
LBIAA	RAL	EZ001		1529	65	0102	0307
	BTU	W0002		0307	20	1968	1513
	RAL	EZ026		1513	65	0675	1511
	STL	W0003		1511	20	1969	0371
	RAL	EZ015		0371	65	0224	0379
	STL	W0004		0379	20	1970	0272
	RAL	EZ016		0272	65	0325	0429

	STL	W0005		0429	20	19771	0373
	RAL	EZ017		0373	65	0326	0681
	STL	W0006		0681	20	1972	0274
	RAL	EZ004		0274	65	0176	0731
	STL	W0007		0731	20	0773	0375
	RAL	EZ005		0375	65	0277	0781
	STL	W0008		0781	20	1974	0376
	RAL	EZ018		0376	65	0479	0333
ES000	LOO	LBKAA	E00AH	0333	65	0286	1907
LBKAA	OO	00000	LBKAA	00000	00	0000	0000
	AXA	Y0000		0286	50	0001	0292
	RSL	H0005		0292	66	8005	0349
	STO	Y0032		0349	24	0033	0336
	ALO	Y0044		0336	15	0025	0529
	BMI	LBKAA	ETO33	0529	46	0232	0136
ES033	OO	00000	LBKAA	00000	00	0000	0303
LBKAA	RAL	Y0044		0232	65	0025	0579
	STL	W0001		0579	20	1967	0420
	RAL	EZ014	LBIAE	0420	65	0276	0831
LBKAI	RAU	Y0029	LBIAF	0554	60	0030	0285
LBKAE	STL	ACC	LBKAO	0831	20	0000	0554
LBKAF	LOO		E00AL	0285	69	0238	1850
	STU	ACC		0238	21	0000	0553
	RAL	W0001		0553	65	1967	0421
	LOO		E00AE	0421	69	0324	1926
	STU	W0001		0324	21	1967	0470
	RAU	ACC		0470	60	0000	0405
	FOV	W0001		0405	34	1967	0437
	RSU	8003		0437	61	8003	0425
	FAO	Y0030		0425	32	0031	0357
	LOO		E00AA	0357	69	0360	1700
	BTU	Y0030	LBMAA	0360	21	0031	0354
ES034	OO	00000		0000	00	0000	0304
LBMAA	RAL	Y0044		0384	65	0025	0629
	STL	W0001	LBMAE	0629	20	1967	0520
	RAL	EZ014	LBMAF	0520	65	0276	0831
LBMAU	RAU	Y0027	LBMAO	0604	60	0028	0383
LBMAE	STL	ACC	E00AL	0881	20	0000	0604
LBMAF	LOO			0383	69	0386	1850
	STU	ACC		0386	21	0000	0603
	RAL	W0001		0603	65	1967	0471
	LOO		E00AE	0471	69	0374	1926
	BTU	W0001		0374	21	1967	0570
	RAU	ACC		0570	60	0000	0455
	FOV	W0001		0455	34	1967	0467
	RSU	8003		0467	61	8003	0475
	FAO	Y0028		0475	32	0029	0505
	LOO		E00AA	0505	69	0358	1700
	BTU	Y0028	LBMAA	0358	21	0029	0282
ES035	OO	00000	LBMAA	0000	00	0000	0305
LBMAA	RAL	Y0044		0282	65	0025	0679
	STL	W0001		0679	20	1967	0620
	RAU	Y0027		0620	60	0028	0437
	FMP	Y0029		0437	39	0030	0230
	STU	ACC		0230	21	0000	0653
	RAL	W0001	E00AE	0653	65	1967	0521
	STU	W0001		0521	69	0424	1926
	RAU	ACC		0424	21	1967	0670
	FOV	W0001		0670	60	0000	0555
	RSU	8003		0555	34	1967	0517
	FAO	Y0031		0517	61	8003	0525
	LOO		E00AA	0525	32	0032	0209
	BTU	Y0031	LB0AA	0209	69	0312	1700
ES036	OO	00000	LB0AA	0312	21	0032	0335
LB0AA	RAU	Y0031		0000	00	0000	0306
	FOV	Y0048		0335	60	0032	0483
	LOO		E00AA	0483	34	0029	0332
	STU	Y0047	LBPA A	0332	69	0385	1700
ES037	OO	00000	LBPA A	0385	21	0048	0151
LBPA A	RAL	Y0044	LBPA C	0000	00	0000	0307
LBPA B	RAU	Y0027	LBPA O	0151	65	0025	0729
LBPA C	LOO	LRPAB	E00AF	0654	60	0028	0533
LBPA U	FOV	ACC		0729	69	0654	1752
	STU	Y0048	E00AA	0533	34	0000	0400
	LOO		LBQAA	0400	69	0703	1700
	BTU	Y0048	LBQAA	0703	21	0049	0202
ES038	OO	00000	LBQAA	0000	00	0000	0308
LBQAA	RAL	Y0024	LBQAC	0202	65	0025	0779
LBQAC	RAU	Y0029	LBQAO	0779	60	0030	0435
LBQAC	LOO	LRQAB	E00AF	0779	69	0704	1752
LBQAH	FOV	ACC		0435	34	0000	0450
	LOO		E00AA	0450	69	0753	1700
	STU	Y0049	LBRAA	0753	21	0050	0803
ES039	OO	00000	LBRAA	0000	00	0000	0309
LBRAA	RAU	Y0048		0803	60	0049	0853
	FMP	Y0047		0853	39	0048	0298
	RSU	8003		0298	61	8003	0605
	FAO	Y0049		0605	38	0050	0627
	LOO		E00AA	0627	69	0280	1700
	BTU	Y0050	LBRAA	0280	21	0051	0754
ES040	OO	00000	LBRAA	0000	00	0000	0400
LBRAA	RSU	EZ013	LBRAU	0754	61	0387	0491



LBSAH	RAL	Y0024	EO0AE	0804	65	0025	0R29
	LOO	LRSAE		0829	69	0382	1926
LBSAH	STU	ACC	LBSAB	0491	21	0000	0804
LBSAE	FAO	ACC		0382	32	0000	0677
	STU	W0001		0677	21	1927	0720
	RAU	Y0031		0720	60	0032	0437
	FHO	Y0047		0437	39	0048	0348
	RSH	R0003		048	61	8003	0655
	FAO	Y0030		0655	32	0031	0407
	FOV	W0001		0407	34	1927	0567
	LOO		EO0AA	0567	69	0770	1700
	STU	Y0051		0770	21	0052	1502
	RAU	03H7		1502	60	03R7	1512
	FOV	Y0051		1512	34	0052	1520
	STU	Y0052	LBTAA	1520	21	0053	0705
ES042	OO	0000	LBTAA	0000	00	0000	0402
LBTA	RAU	Y0050		0705	60	0051	0755
	LOO		EO0BT	0755	69	0051	0104
	STU	Y0050	LSUAA	0163	21	0051	0854
ES043	OO	0000		0000	00	0000	0403
LBAA	RAU	Y0047	EO0BT	0854	60	0048	0903
	STU	Y0047	LBVAA	0903	69	0259	0104
	RAU	Y0051	LBVAA	0259	21	0048	0201
ES044	OO	0000		0000	00	0000	0404
LBAA	RAU	Y0051		0201	60	0052	0457
	LOO		EO0BT	0457	69	0263	0104
	STU	Y0051	LBWAA	0263	21	0052	0305
	RAU	Y0052	LBWAA	0000	00	0000	0405
ES045	OO	0000		0805	60	0053	0507
LBAA	RAU	Y0052	EO0BT	0507	69	0313	0104
	LOO		LBXAA	0313	21	0053	0156
	STU	Y0052	LBXAA	0000	00	0000	0406
ES046	OO	0000		0156	65	0309	0705
LBAA	RAL	EZ019		0363	20	1968	0571
	RAL	EZ020		0571	65	0474	0879
	RAL	EZ003		0879	20	1979	0322
	RAL	EZ021		0322	65	0575	0929
	STL	W0004		0929	20	1970	0423
	RAL	EZ022		0423	65	0426	0931
	STL	W0005		0931	20	1971	0524
	RAL	EZ006		0524	65	0175	0979
	STL	W0006		0979	20	1972	0625
	RAL	EZ004		0625	65	0176	0981
	STL	W0007		0981	20	1973	0476
	RAL	EZ005		0476	65	0277	1031
	STL	W0008		1031	20	1974	0787
	RAL	EZ023		0787	65	0330	0485
	LOO	LBVAA	EO0AR	0485	69	0288	1907
	RAU	Y0000	LBVAA	0000	00	0000	0000
ES000	OO	0000		0288	65	0541	0495
LBAA	RAL	EZ024		0495	20	1968	0621
	STL	W0002		0621	65	0574	1029
	RAL	EZ025		1029	20	1969	0372
	STL	W0003		0372	65	0675	1079
	RAL	EZ026		1079	20	1970	0473
	STL	W0004		0473	65	0526	1081
	RAL	EZ027		1081	20	1971	0624
	STL	W0005		0624	65	0777	1131
	RAL	EZ028		1131	20	1972	0725
	STL	W0006		0725	65	0288	0583
	RAL	EZ029		0583	20	1973	0576
	STL	W0007		0576	65	1129	0633
	RAL	EZ030		0633	20	1974	0827
	STL	W0008		0827	65	0360	0535
	RAL	EZ031		0535	69	0338	1907
	LOO	LBZAA	EO0AR	0000	00	0000	0408
	RAU	Y0000	LBZAA	0338	00	8000	8000
ES048	NOF	R0000		0380	00	0007	0000
LBZAA	OO	0007		1129	00	0000	0027
EZ031	OO	0000		0228	00	0000	0044
EZ030	OO	0000		0777	00	0000	0028
EZ029	OO	0000		0526	00	0000	0029
EZ028	OO	0000		0675	00	0000	0041
EZ027	OO	0000		0574	00	0000	0030
EZ026	OO	0000		0541	00	0000	0031
EZ025	OO	0000		0330	00	0000	0046
EZ024	OO	0007		0426	00	0000	0050
EZ023	OO	0007		0575	00	0000	0047
EZ022	OO	0000		0474	00	0000	0051
EZ021	OO	0000		0309	00	0000	0052
EZ020	OO	0000		0479	00	0007	0032
EZ019	OO	0000		0326	00	0000	0032
EZ018	OO	0000		0325	00	0000	0045
EZ017	OO	0000		0224	00	0000	0046
EZ016	OO	0000		0276	00	0000	0002
EZ015	OO	0000		0387	10	0000	0051
EZ014	OO	0000		0186	10	0000	0053
EZ013	OO	0000		0180	00	0000	0006
EZ012	OO	0000		0191	00	0000	0003
EZ011	OO	0000		0174	00	0000	0009
EZ010	OO	0000		0090	00	0003	0019
EZ009	OO	0001		0289	00	0000	0000
EZ008	OO	0003		0128	00	0000	0001
EZ007	OO	0000		0277	00	0000	0022
EZ006	OO	0005		0176	00	0000	0023
EZ005	OO	0000		0175	00	0000	0024
EZ004	OO	0000		0074	00	0000	0025
EZ003	OO	0000		0102	00	0000	0026
EZ002	OO	0000		1991	00	0000	1999
EZ001	BOF						

SOAP  
Program  
for  
 $\Psi$ AREA

Programmed  
by  
Dr. Thomas S. Parker

IBM 650  
Kansas State University

1	HIMES	FOR	MULA	FOR	STABILITY	1	0000	00	0000	00	0
						2	0000	00	0000	00	0
		8LR	0500	1999		3	0000	00	0000	00	0
		RCO	1951			4	0323	70	1951	00	01
		RAU	1954			5	0001	60	1954	00	09
		RAC	0001			6	0009	88	0001	00	15
		STL	SUM			7	0015	20	0019	00	22
		LOO	AYE	FLOPT		8	0022	69	0025	00	28
FIFIV		OO	0000	0055		9	0000	00	0000	00	55
FPTW0		20	0000	0051		10	0010	20	0000	00	51
FIFT3		OO	0053	0000		11	0100	00	0053	00	00
FLOPT		STO	TEMPZ			12	0028	24	0031	00	34
		8MI	NEG			13	0034	46	0037	00	38
		8CT	0000			14	0038	36	0000	00	11
		STL	INDEX			15	0011	20	0065	00	18
		RAL	8003			16	0018	65	8003	00	75
		SRT	0002			17	0075	30	0002	00	81
		SLT	0002			18	0081	35	0002	00	87
		ALO	FIFIV			19	0087	15	0000	00	05
		SLO	INDEX			20	0005	16	0065	00	69
		RAU	8002	TEMPZ		21	0069	60	8002	00	31
NEG		RBU	8003			22	0037	61	8003	00	45
		SCT	0000			23	0045	36	0000	00	17
		STL	INDEX			24	0017	20	0065	00	68
		RAL	8003			25	0068	65	8003	00	125
		SRT	0002			26	0125	30	0002	00	131
		SLT	0002			27	0131	35	0002	00	137
		ALO	FIFIV			28	0137	15	0000	00	05
		SLO	INDEX			29	0055	16	0065	00	119
		RSU	8002	TEMPZ		30	0119	61	8002	00	31
AYE		STU	AY			31	0025	21	0030	00	33
		RAU	1955			32	0033	60	1955	00	59
		LOO	8EE	FLOPT		33	0059	69	0012	00	28
BEE		STU	8E			34	0012	21	0016	00	169
		FAO	AY			35	0169	32	0030	00	07
		STU	ZE	REPET		36	0007	21	0068	00	115
REPET		RCO	1951			37	0115	70	1951	00	51
		RAU	1956			38	0051	60	1956	00	61
		LOO	ETA	FLOPT		39	0061	69	0014	00	28
ETA		F88	ZE			40	0014	33	0062	00	39
		8TU	ETA1			41	0039	21	0044	00	47
		8MC		DONOT		42	0047	49	0150	00	101
		FMP	ETAO			43	0150	39	0003	00	53
		8MI	OPPOS			44	0053	46	0006	00	57
		RAU	ETA1			45	0057	60	0004	00	49
		FAD	ETAO			46	0049	32	0003	00	29
		FOV	FPTW0			47	0029	34	0050	00	200
		RAM	8003			48	0200	67	8003	00	107
		RAU	8002			49	0107	60	8002	00	165
		FAO	SUM			50	0165	32	0019	00	095
		8TU	SUM	DONOT		51	0095	21	0019	00	101
OPPOS		RAU	ETA1			52	0006	60	0044	00	099
		F88	ETAO			53	0099	33	0003	00	079
		STU	OIFF			54	0079	21	0084	00	187
		FOV	FPTW0			55	0187	34	0050	00	250
		RAM	8003			56	0250	67	8003	00	157
		RAU	8002			57	0157	60	8002	00	215
		FAO	SUM			58	0215	32	0019	00	145
		STU	SUM			59	0148	21	0019	00	072
		RAU	ETA1			60	0072	60	0044	00	149
		FMP	ETAO			61	0149	39	0003	00	103
		FOV	OIFF			62	0103	34	0088	00	134
		RAM	8003			63	0134	67	8003	00	141
		RSU	8002			64	0041	61	8002	00	199
		FAD	SUM			65	0199	32	0019	00	195
		STU	SUM	DONOT		66	0195	21	0019	00	101
QONOT		SXC	0002			67	0101	59	0002	00	207
		LOO	ETA1			68	0207	69	0044	00	097
		STO	ETAO			69	0097	24	0003	00	056
		RAU	ZE			70	0056	60	0062	00	067
		FAO	8E			71	0067	32	0062	00	043
		STU	ZE	REPET		72	0043	21	0062	00	115
UNFLO		RAU	SUM			73	0300	60	0019	00	023
		SRT	0002			74	0023	30	0002	00	129
		RAL	8002			75	0129	65	8002	00	237
		SRT	0004			76	0237	30	0004	00	147
		SLO	FIFT3			77	0147	16	0100	00	105
		8MI	NAGFT			78	0105	46	0008	00	109
		LOO	SHIFT			79	0109	69	0118	00	265
		8OA	SHIFT			80	0265	22	0112	00	315
		RAL	SUM			81	0315	65	0019	00	073
		SRT	0002	SHIFT		82	0073	30	0002	00	112
SHIFT		SLT	0000	PUNCH		83	0112	35	0000	00	035
NAGFT		RSL	8002			84	0008	66	8002	00	177
		LOO	SHOFT			85	0117	69	0020	00	123
		SOA	SHOFT			86	0123	22	0020	00	173
		RAL	SUM			87	0173	65	0019	00	223
		SRT	0002	SHOFT		88	0223	30	0002	00	010
SHOFT		SRT	0000	PUNCH		89	0020	30	0000	00	035
PUNCH		STL	1977			90	0035	20	1977	00	080
		PCH	1977			91	0080	71	1977	00	027
		HLT	1111	1111		92	0027	01	1111	1111	1111

EXPLANATION OF PLATE XXVI

Photograph of drop column, corresponding  
to drawing of drop column presented in  
Plate II.

PLATE XXVI



AN INTRODUCTION TO A NEW CONCEPT:  
THE DYNAMIC STABILITY  
OF  
DISKS FREELY DESCENDING IN A FLUID MEDIA

by

BILLY LEE HIMES, SR.

B. S., Kansas State University of  
Agriculture and Applied Science, 1958

---

AN ABSTRACT OF A THESIS

submitted in partial fulfillment of the

requirements for the degree

MASTER OF SCIENCE

Department of Chemical Engineering

KANSAS STATE UNIVERSITY  
OF AGRICULTURE AND APPLIED SCIENCE

1960

## ABSTRACT

This study pertained to the characteristics of the motions of particles falling freely in a fluid.

There were four objectives to this investigation. The first was to refine the mathematical definition of a property of motion of each particle as it freely descended in an infinite expanse of fluid. This property was to be named dynamic stability. The second objective was to actually measure the behavior of the motion of each particle and subsequently calculate values of dynamic stability. The third was to observe and define the type of motion exhibited by each particle falling freely in a fluid. The fourth was to correlate the results of two and three.

The dynamic stabilities were found by dropping a series of particles, one at a time, through a glass column containing triethylene glycol. The descending particles were filmed on moving pictures and, from the individual frames of this film, measurements were made of the heights of the particles at equal increments of time. From these measurements a least squares line was calculated for the path of each particle. The mean deviation of the actual path of the particle from the least squares line gave dynamic unstability; the reciprocal of this quantity was taken as dynamic stability.

These investigations were conducted with disks - approximately one inch in diameter, a quarter inch thick, and of various weights and moments of inertia - descending in triethylene glycol at 88 to 90°F at Reynolds number, based on  $D_s$ , ranging from 78 to 640.

It was found that a disk, in a free-fall descent through triethylene glycol, exhibited one of three postulated types of motion; i. e. stable,

wave I, and wave III. It was further postulated that wave I motion consisted of subtypes; wave I-A and wave I-B. The distinction between these subtypes was shown by a fluctuation in the values of dynamic stability.

A relationship was found between all the postulated types of motion in triethylene glycol and the dynamic stabilities,  $\psi_{y,x}^2$ . Disk 6, exhibiting stable motion, had dynamic stabilities between 11.49 to 14.92 reciprocal cm. Disks postulated as exhibiting wave I-A, I-B, and III motions had ranges of dynamic stabilities of 6.02 to 8.55 reciprocal cm., 4.15 to 5.46 reciprocal cm., and 0.44 to 2.01 reciprocal cm., respectively. Each range of dynamic stabilities, corresponding to a particular type of subtype of motion, was distinct and separate.

From a comparison of the physical properties of the disks with the types of motion assigned each disk it was found that a low dynamic stability was associated with a high moment of inertia relative to weight of the disk. From a comparison of Reynolds numbers for each disk with the types of motion it was found that a Reynolds number, based on  $D_s$ , of less than one hundred was associated with stable motion.

This study shows that dynamic stability of disks is measurable and therefore must exist. The measurable dynamic stabilities also may be utilized to predict the types of motion of disks descending in a fluid media.